Not all transitions are equal: The relationship between inequality of educational opportunities and inequality of educational outcomes *

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Abstract

In a given educational system, students from different socioeconomic backgrounds can experience inequality in terms of opportunities (Inequality of Educational Opportunity, or IEOpp), or in terms of outcomes (Inequality of Educational Outcome, or IEOut). The former can best be studied by examining the effect of family socioeconomic status (SES) on the probability of passing from one level of education to another. The latter can be studied by estimating the effect of family SES on the highest achieved level of education. Mare (1981) showed that IEOut depends, in part, on the distribution of education, whereas IEOpp does not. This finding has been primarily used as an argument for studying IEOpp and not studying IEOut. This paper aims to build on the results obtained by Mare (1981) to show how IEOpp and IEOut are mathematically related to one another, and how this relationship can be used to obtain a more informative description of differences in educational inequality across groups, such as cohorts or gender. Applying the result to cohorts born in the Netherlands for cohorts born between 1894 and 1978 demonstrates how this model can be used to describe the impact of changes in IEOpp, educational expansion, and the disadvantaged position of women regarding IEOut.

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1 Introduction

Social stratification research has long been concerned with the relationship between family socioeconomic status (SES) and offspring’s educational attainment. A strong positive association between the two implies that children with higher SES backgrounds are more likely to achieve higher levels of education than children with lower SES backgrounds. For this reason, the strength of the relationship is often termed ‘educational inequality’. Distinguishing between inequality relating to the process of attaining education and inequality relating to the final outcome makes the concept of educational inequality more precise. Students face a number of opportunities during their educational careers, and can be operationalized as probabilities of passing transitions between different levels of education. As a result of this, Inequality of Educational Opportunities (IEOpp) can be operationalized as the association between family SES and the probability that a student will pass through these transitions. The outcome of the educational process can be operationalized as the highest achieved level of education. This means that, the association between family SES and the highest achieved level of education is an operationalization of Inequality of Educational Outcomes (IEOut). Both IEOpp and IEOut are of substantive interest, with the former describing the process, and the latter the outcome. Moreover the two are related to one another, with the process leading to the outcome. This article will show how this relationship can be used to obtain a more informative description of educational inequality.

Mare (1981) showed how an estimate of IEOut can be derived from a model estimating IEOpps, and that differences in IEOut between cohorts are in part due to differences in the distribution of education. These effects can be considerable, since the distribution of education varies substantially over cohorts. In almost all countries, people born in later cohorts have attained more education, a process that has been termed ‘educational expansion’ (Hout and DiPrete, 2006). Furthermore, Mare (1981) showed the IEOpps control for this effect of educational expansion. This led Mare (1981) to argue that the IEOpp is a more ‘pure’ measure of educational inequality. Since then, the literature has approached the relationship between IEOut, the IEOpps, and the distribution of education as a black box. Those studies
that have investigated the relationship (Mare, 1981; Smith and Cheung, 1986; Nieuwbeerta and Rijken, 1996) compare the observed IEOut with the simulated results of two counterfactual scenarios, those being that either the distribution of education remained unchanged and IEOpp changes as observed; or that the distribution of education changes as observed, but IEOpp remains unchanged. Simulations such as these can tell us how much IEOut is affected by changes in the distribution of education and changes in IEOpps, but do not offer us any insights as to why.

This paper aims to demonstrate that the relationship identified by Mare (1981) has a substantive interpretation, and can be used to obtain a more informative description of educational inequality and, in particular, of differences in educational inequality across groups. The starting point for this enriched description is the realization that the IEOpps (the process) leads to IEOut (the outcome), but that not every IEOpp (that is, every step in the process) is of equal importance for reaching the outcome. Moreover, as will be shown below, the importance of each IEOpp for the IEOut can differ across groups. In most industrialized countries, for instance, virtually all students remain in education after the primary level. As a result, any inequality at this first transition only affects a few (or no) students, and is thus not very important for IEOut. The situation was quite different at the beginning of the twentieth century however: then much more students failed to continue after primary education, so the IEOpp for the transition between primary and secondary education was much more important for the IEOut than it is now. The intuition presented in this example will be further developed this article.

In particular this article will address the following questions:

What is the relationship between IEOpps, IEOut, and the distribution of education?

How can this relationship be used when describing differences between groups in IEOut and IEOpp?

These questions will be answered by expanding on the research conducted by Mare (1981). Mare (1981, pp. 77–78) showed that IEOut is a weighted sum of the IEOpps. These weights
capture the effect of the distribution of education, whereby the distribution of education is represented by the set of probabilities of passing each transition. The model proposed by Mare (1981) for estimating IEOpps implies an IEOut that can be decomposed into the IEOpps and their various weights. This paper will show that each IEOpp’s weight depends on the distribution of education in a substantively interesting way. Namely, an IEOpp receives more weight if: 1) the proportion of people ‘at risk’ of making that transition increases; 2) the proportion passing that transition is closer to 50%, that is, passing or failing that transition is not almost universal; and 3) the difference in expected level of education between those who pass and those who fail to make the transition increases, that is, the gain from passing increases. These weights thus constitute an attractive operationalization of the importance of the IEOpps.

The result proves to be interesting in its own right, as it sheds light on the relationship between inequality relating to the process of obtaining education (IEOpp) and inequality relating to the end result (IEOut). These findings also have important policy implications, suggesting that if one wants to reduce IEOut, then this can be achieved through reducing the IEOpps or by bypassing highly unequal transitions by letting almost everybody pass (or fail) those transitions. This is important because the number of students that pass a transition is more easily influenced by government policy, for instance via subsidies for tuition fees, the provision of scholarships, or — when the number of places is limited — making more places available. Furthermore, the results suggest that even though it depends on the distribution of education, IEOut is a useful and meaningful measure of educational inequality. Finally, the findings enable us to study the effect of educational expansion on IEOut. Educational expansion means that compared over cohorts more students pass the various transitions, and this change in turn leads to changes in the weights assigned to each IEOpp. Educational expansion thus influences IEOut by influencing the importance of each transition. The influence of differences in the distribution of education between men and women or between countries can be studied in the same way, with differences in the distribution of education reflecting gender educational inequality and differences between educational systems.

One should note that educational inequality between persons from different socioeconomic
backgrounds is treated differently from other types of educational inequalities, such as, educational inequality between cohorts or between men and women. This primary form of educational inequality is divided into two constituent parts: IEOpps that do not depend on the distribution of education, and IEOOut that does depend on the distribution of education. As regards other types of inequality this article simply focusses on the fact that the distributions are different, and these other types are thus addressed at a much more general level. To emphasize this, the generic terms ‘gender educational inequality’ and ‘educational expansion’ will be used to refer to the differences in the distribution of education between men and women and between cohorts respectively.

This paper will begin with a description of the model proposed by Mare (1981), and the derivation of the relationship between IEOpp, IEOOut, and the distribution of education. In the next section the result is applied to differences in IEOOut between men and women and across cohorts born in the Netherlands between 1894 and 1978. This case illustrates how this approach can enrich our understanding of educational inequality. This section will also generalize the relationship between IEOOut, IEOpp and the distribution of education to models that are suitable for a tracked educational system.

2 Inequality of educational opportunities and outcomes

IEOpps are the effects of family background on the probabilities that a student will pass the different transitions between levels of education. These effects are typically estimated using a sequence of logistic regressions. This model is known by a variety of names, including the sequential response model (Maddala, 1983), the continuation ratio logit (Agresti, 2002), the model for nested dichotomies (Fox, 1997), and the Mare model (Shavit and Blossfeld, 1993). This paper will use the term ‘sequential response model’ or, ‘sequential logit model’ to emphasize that logistic regression is used to model the probabilities of passing transitions.

The aim of using a sequential logit model is to enable us to study transitions. Frequently, however, the data used only show the highest achieved level of education (see, for example Shavit and Blossfeld, 1993). The solution is to assume that each level of education can
be achieved through one and only one path through the education system. Thus knowing someone’s highest achieved level of education implies that one knows which transitions a person passed through. Consider, for instance, a hypothetical education system consisting of four levels: no education, primary education, secondary education, and tertiary education. Figure 1 shows how children face three transitions in this system: they can attend primary education or opt for no education at all; if they opt for primary education they can choose to leave the system once they have completed primary education, or go on to secondary education; and if they opt for secondary education, they can then either choose to leave once they have completed this level or go on to tertiary education. The implication is that if someone’s highest-achieved level of education is primary education, then that person was ‘at risk’ of passing through the first two transitions, but not the third. Furthermore, it implies that the person passed the first transition, but failed the second. As this is a simple and commonly used model, it will be used to introduce the relationship between the IEOpps and IEOut (see, for example Shavit and Blossfeld, 1993). It does not offer a good description of a large number of education systems around the world, however, especially those consisting of multiple tracks. For this reason, the results from this section will be generalized into a model that is more appropriate for a tracked system. This will be illustrated by applying the generalized model to the Netherlands, a country with a tracked education system.

The simple model assumes that, one has to be at risk of passing a transition — that is, to have passed through all lower transitions — in order to make a decision at that transition about whether to continue in education or to leave the system. Aside from this, these decisions are assumed to be completely independent. As a result one can estimate the IEOpp by running separate logistic regressions for each transition on the appropriate sub-sample. This model is shown in equation (1).

\[
p_{ki} = \frac{\exp(\alpha_k + \lambda_k SES_i)}{1 + \exp(\alpha_k + \lambda_k SES_i)} \quad \text{if} \quad pass_{k-1i} = 1
\]

The probability that person \(i\) passes transition \(k\) is \(p_{ki}\). The IEOpp belonging to transition
$k$ is $\lambda_k$ and the constant for transition $k$ is $\alpha_k$. Whether or not individual $i$ has passed the previous transition is indicated by the indicator variable $\text{pass}_{k-1,i}$. It is assumed that everybody is at risk of passing the first transition, meaning that $\text{pass}_{0,i} = 1$. The differences in IEOpp between men, women, and cohorts can be obtained by adding the appropriate interaction terms to the model.

In order to make a link between the IEOpps (the $\lambda_k$s) and IEOut, it is necessary to assign a value ($l_k$) to each level of education. Commonly used values are based on years of education, and the values used in this example are quite typical for such a metric. This is not the only possible scaling, however, and an alternative metric will be used in the empirical application. A scaling of education makes it possible to use the sequential logit model to calculate the expected highest achieved level of education ($E(L)$). Once a sequential logit model has been estimated, it is then a straightforward process to compute predicted probabilities for passing each transition. The expected highest achieved level of education is the sum of the value of each level of education times the probability of attaining that level, as set out in equation (2). The probabilities and levels can be derived from figure 1. Note that a family’s $SES$ is part of equation (2), through the $p_{ki}$s described in equation (1). Equation (2) can be understood as a regression equation showing a non-linear relationship between a family’s $SES$ and the highest achieved level of education. Using a sequential logit model to derive such a (non-linear) regression is unusual, but has the advantage of being able to describe the IEOpps, the IEOut, and the relationship between them. A more common method is to use a linear regression of highest achieved level of education on family $SES$ (see, for example, Blau and Duncan, 1967; Shavit and Blossfeld, 1993). Linear regression can be understood as a linear approximation of the non-linear relationship implied by the sequential logit model. How well this approximation works is an empirical question.

$$E(L) = (1 - p_{11})l_0 + p_{11}(1 - p_{21})l_1 + p_{11}p_{21}(1 - p_{31})l_2 + p_{11}p_{21}p_{31}l_3$$  \hspace{1cm} (2)

Recall that IEOut is the effect of a family’s $SES$ on the respondent’s highest achieved level of education, or, in other words, how much the expected highest achieved level of education
changes if a family’s \(SES\) changes. Consequently, \(IEOut\) is the first derivative of equation (2) with respect to a family’s \(SES\). This derivative is shown in equation (3). A step-by-step derivation is set out in appendix A.

\[
\frac{\partial E(L)}{\partial SES} = \{1 \times p_1(1 - p_1) \times [(1 - p_2)l_1 + p_2(1 - p_3)l_2 + p_2p_3l_3 - l_0] \} \lambda_1 + \{p_1 \times p_2(1 - p_2) \times [(1 - p_3)l_2 + p_3l_3 - l_1] \} \lambda_2 + \{p_1p_2 \times p_3(1 - p_3) \times [(l_3 - l_2)] \} \lambda_3
\]

Equation (3) shows that \(IEOut\) is a weighted sum of the \(IEOpps\) (the \(\lambda_k\)s). The weights (the sections between curly brackets) consist of three parts all of which are related to the distribution of education. These are:

1. The proportion of people at risk if passing a transition. For the first transition, this proportion is 1; for the second it, is the proportion of students who complete primary education, \(p_1\); and for the third transition, it is the proportion who completed secondary education, \(p_1p_2\). Substantively, this means that a transition is more important when more people are at risk of passing it.

2. The variance of the indicator variable showing who passed and who failed the transition, \(p_k(1 - p_k)\). This variance is a function of the proportion passing. This is lowest if virtually everybody passes or fails, and is highest when the probability of passing is .5. This makes sense at a Substantive level, because if only a few people pass or fail a transition, then any inequality at this stage will only affect a few people.

3. The differences between the expected level of education of those who pass the transitions and those who do not. These are the parts in the square brackets. For instance, the expected level of education of those who pass the first transition is \((1 - p_2)l_1 + p_2(1 - p_3)l_2 + p_2p_3l_3\) and the expected level of education for those that fail the first transition is \(l_0\). The difference between the two is the expected gain from passing the first transition. The substantive interpretation of this is that a transition becomes more important if
passing it leads to a greater expected increase in the highest achieved level of education.

The result is summarized below. IEOut is a weighted sum of IEOpps, and the weights are the product of the proportion at risk, the variance, and the expected gain in level of education resulting from passing.

\[
\text{IEOut} = \sum_{k=1}^{K} (\text{weight}_k \times \text{IEOpp}_k)
\]

weight\(_k\) = at risk\(_k\) × variance\(_k\) × gain\(_k\)

One of the main advantages of this result is that it allows a decomposition of differences between groups in IEOut into differences in IEOpps, and differences in the importance of each transition. These differences in importance can, in turn, be explained by differences in the proportion of people at risk at each transition, the universality of passing or failing, and the gain to be achieved from passing. This makes it possible to study the influence of factors such as educational expansion on IEOut. Educational expansion influences the weight assigned to each transition, because it implies differences in the proportion of people facing each transition, how universal passing or failing is, and the gain to be achieved from passing. It thus provides a substantively interpretable link between the IEOpps, educational expansion, and IEOut. Once can easily extend this argument, allowing us to study the roles played by of gender educational inequality, racial educational inequality, or differences in the distribution of education between countries. One should note that this requires nothing more than a different representation of the results from a sequential logit model and a scaling of the levels of education. The model directly produces estimates of the IEOpps, and can be used to predict the probabilities of passing transitions. In turn, this can be used together with the scaling to calculate the weights and components according to equation (3). A graphical representation of this information is presented during the empirical discussion.

This decomposition depends on the values of all the explanatory variables. One should note that the weights depend on the probabilities of passing the various transitions, and these probabilities in turn depend on all the explanatory variables through equation (1).
Throughout this article IEOut will be evaluated at the mean values of the explanatory variables.

The model discussed thus far is an extension of the research carried out by Mare (1981). The model has, however, been subject to an influential critique by Cameron and Heckman (1998), and in order to understand one of this critique’s key points, it is helpful to make a further distinction between different types of inequality. Consider IEOut: in this case, inequality could refer to a comparison between the mean highest achieved level of education of a group of higher status people and the mean highest achieved level of education of a group of lower status people. Inequality could also refer, however, to a comparison between the expected highest achieved level of education of a low status person, and the expected highest achieved level of education of that same person, had that person had higher status parents. One type of inequality thus refers to the effect on groups, while the other refers to the effect on individuals. Models estimating the former type of inequality are often called marginal or population average models, while models estimating the latter type of inequality are called conditional (on all observed and unobserved variables) or causal models (see, for example, Fitzmaurice, Laird and Ware, 2004; Agresti, 2002). Population average models are appropriate for answering questions about group-level processes, such as the role of education in the reproduction of social classes and how social groups differ with respect to the average level of their members’ education. Causal models are more appropriate for the study of individual-level processes, such as how someone’s education is influenced by their socioeconomic background. Cameron and Heckman (1998) criticize the common interpretation of sequential logit models as causal models. They, and others (for example, Allison, 1999; Mare, 1993), show that sequential logit models are not a causal models, but instead population average models. This means that care must be taken to interpret the results from a sequential logit model in terms of inequality of groups, and not in terms of causal effects on individuals. Alternatively, one could try to estimate a causal model. Such models are harder to identify, however, as they involve making a counterfactual comparison, that is, what would have happened had a person with low status parents had high status parents.
3 Empirical application

This section will illustrate how the relationship between the distribution of education, the IEOpp and IEOut can be used to gain a more complete picture of educational inequality. In particular, this section will describe the influence of educational expansion and gender inequality on status IEOut in the Netherlands for cohorts born between 1894 and 1978. This section will also generalize the previous section’s results in two ways. The first generalization concerns the way in which the sequential logit was parameterized as a series of decisions between staying in and leaving the education system whereby none of the decisions involved a choice between tracks. This corresponds to the model originally proposed by Mare (1981) and remains the most common parameterization. Many education systems do contain tracks, however, and this parameterization does not describe these systems well. Alternative parameterizations of the sequential logit model have been proposed that are more appropriate for tracked systems (Lucas, 2001; Breen and Jonsson, 2000), and I will show that the relationship between IEOpps and IEOut from the previous section also holds for these alternatives. As an illustration, the method is applied the case of the Netherlands, a country with a tracked education system. The second generalization concerns the way in which education is scaled. In the example, the levels of education differ with respect to the amount of investment needed to attain them. They can also differ with respect to how much a student can expect to gain from achieving a certain level, and this is illustrated by using another scale of education.

This section begins by describing the Dutch education system, and presents the data. Next, the results from the sequential logit model discussed in the previous section are generalized so as to make them more applicable to tracked education systems. The IEOpps and IEOut are measured as the strength of the effect of the father’s occupational status on the probabilities of passing the different transitions and the highest achieved level of education respectively. The father’s occupational status was chosen because data was available for this variable over a much longer period of time than for common alternatives, such as the mother’s occupational status, or the father’s and/or mother’s level of education. Finally, the model will be estimated using Dutch data, and the results are presented.
3.1 The Dutch education system

Like many others, the Dutch education system is a tracked system. Figure 2 shows the education system as it was formalized in 1968. A child finishes LO (primary education) when he or she is about 12 years old, and can then choose between four levels: LBO (junior vocational education), MAVO (junior general secondary education), HAVO (senior general secondary education), and VWO (pre-university education). LBO and MAVO both give access to MBO (senior secondary vocational education). HAVO gives access to HBO (higher professional education), and VWO gives access to WO (university). To make things even more complicated, students can also choose to ‘move up’ within their current column (LBO to MAVO, MBO to HBO, and so forth), or ‘move down’ in the next column (HAVO to MBO, and VWO to HBO).

![Figure 2 about here.]

The Dutch education system changed over the period studied. One important development during this period was the emergence of MBO. Initially this was a very heterogeneous level of education, and only slowly gained popularity. This, in turn, had an impact on the nature of MAVO, the primary purpose of which gradually changed from preparing students for the labour market to preparing them for MBO. Another differences was that prior to 1968, the equivalent of HAVO was reserved for girls. Moreover, at the beginning of the period studied, women had only just been granted full access to VWO: prior to 1906, women had to ask the Minister of Education for permission to enter VWO (van Essen, 1990). Table 1 sets out the levels of education, the number of years of education they represent, their English equivalents, and their ISCED classification (UNESCO, 1997).

![Table 1 about here.]

3.2 Data

The data were obtained from the International Stratification and Mobility File (ISMF) (Ganzeboom, 2005). The ISMF has 51 surveys on the Netherland, carried out between 1958
and 2004. These were merged to increase the time period covered and the number of respondents, and to lessen the effect individual surveys’ idiosyncrasies. The purpose of this paper is to compare the effect of a family’s SES on the highest achieved level of education and on probabilities of passing transitions, both between men and women and across cohorts. Time was measured by annual birth-cohorts, measured in decades since 1900, and information was available for the cohorts born between 1894 and 1978. A family’s SES was measured according to the father’s score on the International Socio-Economic Index of occupational status (ISEI) (Ganzeboom and Treiman, 2003), as this measure was available for the largest number of cohorts. The original ISEI score is a continuous variable ranging from 10 to 90, but it was standardized to have a mean of 0 and a standard deviation of 1 for the cohort born in 1940 (approximately the middle of the time period covered in this study) Survey weights were used where available. The weighted number of respondents was approximately 77,000, and after removing respondents with missing observations on any of the variables, approximately 67,000 respondents remained. The number of respondents was unequally distributed over the cohorts, ranging from 47 observations in 1900, to 2,476 in 1947, to 205 in 1976.

Finally, a scale for the level of education was needed in order to estimate the relationship between partial and IEOut using equation (3). The scaling of education used in this paper was estimated in such a way that it maximized the direct effect of education on income controlling for the father’s occupational status. For interpretability this scale was coded in such a way that the mean was 0 and the variance was 1 for the cohort born in 1940 (for details, see appendix B). Whereas in section 2, education was scaled according to the number of years of education, for the empirical application the estimated scale was used. The difference between the latter and the metric in terms of years of education is that the latter defines levels of education in terms of how much a student will gain from achieving a level of education, instead of defining it in terms of how much was invested to achieve a particular level. This was done in order to illustrate that it is possible to treat education as either an investment or as a resource. Which one is chosen depends on the question one wants to answer.

Various Multiple Imputation models (Little and Rubin, 2002) were tried and none of them changed the conclusions.
3.3 Generalizing the sequential response model to a tracked system

Thus far, the sequential response model has been discussed in terms of the simple education system represented in figure 1. Although, as we have seen, the Dutch education system is complex, the system was simplified for estimation purposes (see: figure 3). Two particular simplifications were made: First, some levels were merged (LBO and MAVO, HAVO and VWO and, HBO and WO); and second, it was assumed that each level of education could be attained via just one sequence of transitions. This latter assumption is necessary if one wants to use data containing only the highest achieved level of education. It is this assumption that makes it possible to reconstruct for each transition whether a respondent was at risk of passing that transition, and if so, whether the child passed or failed. ² The simplified representation of the Dutch education system assumes that all children complete primary education. After this, they face a choice between leaving the schooling system and continuing. If they opt for the latter choice, they have to choose between HAVO/VWO (the ‘high track’) and LBO/MAVO (the ‘low track’). Once they have finished their second diploma in either track they can choose whether or not to get a third diploma, continuing with: MBO (if they are in the low track), or HBO/WO (if they are in the high track). The assumption that all children complete primary education can be justified on the ground that the data only shows a very small proportion of people with no primary education. Moreover, completing at least six years of education was compulsory over the entire period studied. The simplified model also implies that each student remains in his or her track. The fact that it is possible to switch between tracks would suggest that this assumption might not always hold true. Three of the 51 surveys used in this study contain information about the actual transitions experienced by respondents, and these show that only a small proportion of all moves between levels of education that could involve crossing the line between the ‘low’ and the ‘high’ tracks actually do cross that line. The percentages of moves that start in the low track and end in the high track or vice versa are 5.7%, 7.6%, 8.7%, and 8.9% for cohorts born between 1927-1932, 1933-1947, 1948-1962, and 1963-1972 respectively. Hence the assumption that people stay within their track seems

²It is possible to relax this assumption if one has data on the actual educational careers. However, data on the highest achieved level of education is typically available for more cohorts than data on complete educational careers and is thus more suitable for studying long term trends.
As before, logistic regressions were used to model the probabilities of passing the different transitions. However, whereas a conventional sequential response model consists of a sequence of decisions to either continue or to stop, this model also contains a ‘branching point’, or a choice between tracks. In this sense the model is akin to those proposed by Lucas (2001) and Breen and Jonsson (2000). Again, the IEOpp and the predicted probabilities belonging to transition $k$ are represented by $\lambda_k$ and $p_{ki}$ respectively. The predicted level of education is now represented by equation (4).

$$\begin{align*}
E(L) &= (1-p_{i1})l_1 + p_{i1}(1-p_{2i})(1-p_{3i})l_2 + p_{i1}(1-p_{2i})p_{3i}l_3 + p_{i1}p_{2i}(1-p_{4i})l_4 + p_{i1}p_{2i}p_{4i}l_5 \tag{4}
\end{align*}$$

Recall that the IEOOut is first derivative of equation (4) with respect to a family’s SES. This derivative is shown in equation (5).

$$\begin{align*}
\frac{\partial E(L)}{\partial \text{SES}} &= \{1 \times p_1(1-p_1) \times [(1-p_2)(1-p_3)]l_2 + (1-p_4)p_{3i}l_3 + p_2(1-p_4)l_4 + p_2p_4l_5 - l_1 \} \lambda_1 + \\
&\{p_1 \times p_2(1-p_2) \times [(1-p_4)]l_4 + p_4l_5 - (1-p_3)l_2 - p_3l_3 \} \lambda_2 + \\
&\{p_1(1-p_2) \times p_3(1-p_3) \times [(l_3-l_2)] \} \lambda_3 + \\
&\{p_1p_2 \times p_4(1-p_4) \times [(l_5-l_4)] \} \lambda_4 \tag{5}
\end{align*}$$

Just as with the example described in section 2, IEOOut is a weighted sum of the IEOpps, the $\lambda_k$s. The weights (the parts between curly brackets) consist of the same three parts:

1. The proportion of people at risk. This is defined slightly differently for the third and fourth transition, however. For the third transition, it is defined as the proportion of
students who continued with the lower track after primary education, \( p_1(1 - p_2) \); and for the fourth transition it is defined as the proportion of students who continued with the high track after primary education, \( p_1p_2 \).

2. A part \( (p_k(1 - p_k)) \) that is small if virtually everybody passes or fails that transition and is largest when the probability of passing is 0.5.

3. The differences between the in expected levels of education of those who pass the transitions and those who do not (these are the parts in the square brackets).

This case illustrates that the relationship between IEOut and IEOpp can be extended to tracked education systems. Using the same logic the result can be extended to even more complex systems, such as those with more than two tracks. In this case a multinomial logit would be used to estimate the IEOpp. However, if one wants to use data with only the highest achieved level of education, one must ensure that for these more complicated systems, each level can only be reached through one — and only one — path through the education system. The decomposition of IEOut into IEOpps and weights for such general education systems has been implemented in Stata in the \texttt{seqlogit} package (Buis, 2007).

### 3.4 Results

The following analysis consists of three parts. First, a descriptive analysis is performed on the differences in transition probabilities between men and women, and between cohorts. Second, the sequential response model described in the previous section is estimated. The results from this model are used to compute the IEOpps, the weights and the IEOut. Together these provide a detailed picture of status educational inequality and how it is influenced by educational expansion and gender inequality. Third, the relationship between the transition probabilities and the weights is investigated in more detail by looking at the three components of the weights: the proportion at risk, the closeness of the transition probability to 50%, and the expected increase in the level of education when passing a transition.

The distribution of the highest achieved level of education is shown in figure 4, for both males and females and for different cohorts. The changes over cohorts were smoothed by show-
ing the predicted proportions from a multinomial logit model explaining highest achieved level of education, with cohort measured as a restricted cubic spline (Royston and Parmar, 2002, Appendix 2), with knots at 1900, 1920, 1940, 1950, and 1970 estimated separately for men and women. A restricted cubic spline means that the trend is restricted to being linear before the first knot (1900) and after the last knot (1970). This restriction was used because when using an unrestricted cubic spline the trend tends to be too erratic near the beginning and end of the curve. The restriction tends to stabilize the curve. As with most other countries, the Netherlands experienced a period of educational expansion during the twentieth century. The proportion of pupils who only achieved LO (primary education) dropped dramatically, while the proportion attaining HBO/WO (higher professional and university) education and MBO (higher secondary vocational) strongly increased. Figure 4 also shows that MBO is a recent level of education. Whereas no one from the earlier cohorts completed this level of education, MBO completion has rapidly grown to about 40%. Furthermore, women experienced all of these developments later than men.

To investigate the IEOpps and IEOOut and how they are influenced by gender educational inequality and educational expansion (differences in the distribution of education between men and women and between cohorts respectively), sequential logit models were estimated separately for both men and women. This is equivalent to adding interaction terms between gender and all other variables. The other variables are: cohort measured as a restricted cubic spline with knots at 1920, 1950, and 1970; the father’s occupational status; and an interaction term with cohort. A model with a non-linear interaction between the father’s occupational status and the cohort was also estimated using the same restricted cubic spline as the main effect of cohort, but the non-linear terms proved to be non-significant ($\chi^2 \text{ 4.73 with 4 df for men and }\chi^2 \text{ 5.50 with 4 df for women}$). The effects are log odds ratios. The main effects of the father’s occupational status are the IEOpps for the cohort born in 1900. This shows that the IEOpps for the higher transitions (in particular LBO/MAVO versus MBO and HAVO/VWO versus HBO/WO) are smaller than for the the lower transitions.
This pattern has also been found by many other studies using sequential response models (Mare, 1980; Shavit and Blossfeld, 1993). Two explanations are commonly given for this. First, children passing the higher transitions are on average older than children passing the lower transitions, and older children are less likely to be influenced by their parents than younger children (Shavit and Blossfeld, 1993). Second, selection on unobserved variables is likely to induce a negative correlation between the observed and unobserved variables, thus suppressing the effect of the observed variables (Mare, 1981) (although Cameron and Heckman (1998) show that this is not always the case). The interaction terms represent the change in effect for every ten-year change in cohort. These show that the effect of the father’s occupational status changed most for the first transition. For men, this is the only transition in which the EIOpp changed significantly over cohorts. This pattern has already been found in the Netherlands (De Graaf and Ganzeboom, 1993), and is being found more frequently in studies of other countries (Breen and Jonsson, 2005).

From these results, one can derive predicted levels of education for each level of the father’s occupational status, forming a non-linear regression line. The slope of this regression line will reveal how much the expected level of education changes when the father’s occupational status changes by one unit, thus providing the IEOut. Figure 5 presents these lines for three cohorts (1900, 1935, and 1970), and for men and women. This figure shows that in all cases, having a father with a higher socioeconomic status will lead to a higher expected level of education. Also, it shows that while women initially suffered a disadvantage, they have recently overtaken men. Finally, the results show that for the earliest cohort, the inequality of educational outcomes for a child with a typical background\(^3\) was relatively small (the curve

---

\(^3\)The father’s occupational status is standardized, so a child with a typical background has a father’s status of 0. Note, however, that the standardization uses the cohort born in 1940, and the average of the father’s status increased over cohorts. The average of father’s occupational status remained reasonably constant until about 1930 at about -0.2 and then steadily increased to 0.5. These changes not only reflect changes in economic structure, but also change in the difference in the number of children between higher and lower status fathers. Consequently, it is hard to give a substantive interpretation to these changes. To simplify the analysis, a child with a typical background will be fixed at the typical background (average father’s occupational status) for a typical cohort (1940).
is rather flat), because everybody in the child’s immediate neighbourhood had an expected level of education that was close to the minimum. However, in this same cohort, very high status children do a lot better than the other children, which would lead to a high inequality of educational outcome. This suggests yet another distinction between types of inequality: between local inequality, which compares a child from a typical background with his or her immediate neighbours, and global inequality, which looks at what happens over the entire curve. One should note, however, that the two types of inequality are the same when the curve is linear, as is approximately the case for the cohorts from 1935 and 1970. To simplify the analysis, the term IEOut is used throughout this paper to refer to local IEOut. One should also note that the local and global IEOpps, as measured by the log odds ratio of passing transitions, are assumed to be the same, since the effect of the father’s occupational status is assumed to be linear.

Figure 6 shows the IEOut according to the sequential response model. Both education and the father’s occupational status are scaled in such a way that the mean for the cohort 1940 is 0 and the standard deviation is 1. So this measure of IEOut is similar to a standardized regression coefficient. IEOut displays two striking features: the first is the trend in IEOut, which initially increases and then decreases. As will be shown later, both the initial increase and the later decrease can be explained by the phenomenon of educational expansion. The second feature is the consistently lower IEOut for women. For most cohorts, this is the result of gender educational inequality, and only for the most recent cohorts is this due to differences in IEOpps.

This pattern in IEOut can be explained by looking at the differences in transition probabilities between men and women and cohorts, as these probabilities determine how much each transition contributes to IEOut. Recall that each transition’s contribution is the weight times the IEOpp. This means that each transition’s contribution can be visualized as the area of
a rectangle, with a height equal to the IEOpp and a length equal to the weight. For men and women, this is set out in figures 7 and 8. The horizontal axis shows the weights and the vertical axis the IEOpp, whereas the columns represent the cohorts and the rows represent the transitions. The IEOOut for a cohort (as shown in figure 6) is the sum of the areas of the rectangles within one column. In this way one can compare the relative contribution of the different transitions to IEOOut, and ascertain how these differences are caused by differences in IEOpp and their weights.

Figures 7 and 8 show that the first two transitions are the main sources of IEOOut are the first two transitions. The main change over time is the replacement of the first transition (continue after LO) by the second transition (LBO/MAVO versus HAVO/VWO) as the dominant contributor to IEOOut. Closer inspection shows that initially, both the first transition’s contribution and that of the second transition increase, thereby causing the initial increase in IEOOut. Later, IEOOut decreases as the first transition starts to contribute less and less to IEOOut. The reason for women’s lower IEOOut changes over the cohorts. Initially, the lower IEOOut for women was due to lower weights and equal (first transition) or higher (second transition) IEOpps. For the later cohorts, the weights were approximately equal between men and women, but the IEOpps were smaller for women.

Figures 9 and 10 show the origin of these changes in the weights. They consist of changes relating to three aspects of the distribution of education: the proportion of people at risk at each transition; the closeness to 50% of the proportion of people passing (the variance); and the difference in the expected level of education between those passing and those failing a transition. All three elements are a function of the proportions that pass each transition, so these are reported in the top panel of figures 9 and 10. The three panels in the model represent the three components of the weight, while the bottom panel represents the weight, which is the product of the three components.

[Figure 7 about here.]

[Figure 8 about here.]

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[Figure 9 about here.]
Figures 9 and 10 show that the initial increase and the later decline in the first transition’s influence is primarily due to the variance. Initially, any inequality at this transition affected few people, because a low proportion passed. As the proportion people passing increased, the transition received more weight, until half of the students passed, after which inequality affected few people because few people failed. The increase in importance of the second transition is partly due to the variance, but also to a strong increase in the number of students that are at risk of making this transition. For women these developments have occurred later, leading to smaller weights. The last two transitions are relatively unimportant, because relatively few people are at risk of passing these transitions, and those who pass gain relatively little. Those who pass the first two transitions gain both the immediate increase in level of education and the probability of gaining an extra level of education (either MBO or HBO/WO), while in the third and fourth transition, people only gain the immediate increase in level of education.

4 Conclusion

This paper began by making a distinction between Inequality of Educational Opportunities (IEOpp) and Inequality of Educational Outcomes (IEOut). This paper then sought to answer the following questions: what is the relationship between IEOpps, IEOut, and the distribution of education, and how can this relationship be used when describing differences between groups in IEOut and IEOpp? It was shown that IEOpps influence IEOut, but that IEOpps from different transitions differ with respect to their importance for IEOut. The importance of an IEOpp depends on the distribution of education. IEOut is shown to be a weighted sum of the IEOpps, and a transition’s IEOpp receives more weight if more people are at risk; if passing or failing the transition is less universal (that is, if the proportion of respondents who pass is closer to 50%); and if there is a larger difference in the expected level of education of people who pass and fail.

These findings are an extension of the research carried out by Mare (1981), who demon-
strated that differences in IEOOut across groups (cohorts, for example) depend on both the differences in IEOOpp and differences in the distribution of education. The literature has treated this relationship as a ‘black box’, however, and as an argument for controlling for the distribution of education rather than of studying its effects. This has proved to be a lost opportunity, since the differences in distribution of education represent substantively interesting phenomena, such as educational expansion, and have important policy implications. One should note that this does not require estimation of a model other than the sequential logit model proposed by Mare (1981). What is proposed here is simply a different representation of the results from such a model.

The empirical application of this finding focused on the impact of differences in the distribution of education across cohorts (educational expansion) and gender (gender educational inequality) on IEOOut in the Netherlands, for cohorts born between 1894 and 1978. These two factors can explain two main features of the trend in IEOOut. First, the trend over cohorts showed an initial increase followed by a decrease. Second, the IEOOut is consistently lower for women. The initial increase in IEOOut can be explained by educational expansion, in particular the increase in the proportion of respondents passing the first and second transition, that is, the transition between primary education and some form of secondary education and the transition between entering LBO/MAVO (the low track) or HAVO/VWO (the high track). Initially, a small proportion of pupils passed these transitions, meaning that this transition made a small contribution to IEOOut. This proportion rose to about 50%, however, greatly increasing these transitions’ contributions and causing a rise in IEOOut. The proportion of students passing the first transition, however, continued to rise rapidly to almost universal passing. As a consequence this transition’s contribution to IEOOut decreased, causing the subsequent decrease in the IEOOut. The reason for women’s lower IEOOut changes over the cohorts. Initially, IEOOut was lower for women because fewer women passed each transition, causing each transition’s weight to be less for women than for men. For the later cohorts, weights were approximately equal between men and women, because women were as likely or even more likely to pass transitions as men. However, for these later cohorts the IEOOpps were smaller for women than for men, causing the IEOOut for women to remain below the IEOOut.
for men.

In conclusion, this paper has shown how the study of educational inequality can be enriched by studying the impact of the distribution of education, rather than by simply controlling for it. This has the key advantage of allowing us to study the impact of phenomena such as educational expansion or the disadvantaged position of women regarding IEOut.

Appendices

A Derivation of equation (3)

Equation (3) is the first derivative of equation (2). Equation (2) is repeated below:

\[ E(L) = (1 - p_{1i})l_0 + p_{1i}(1 - p_{2i})l_1 + p_{1i}p_{2i}(1 - p_{3i})l_2 + p_{1i}p_{2i}p_{3i}l_3 \]

whereby the \( p_{ki} \)s are represented by equation (1), repeated below:

\[ p_{ki} = \frac{\exp(\alpha_k + \lambda_k SES_i)}{1 + \exp(\alpha_k + \lambda_k SES_i)} \text{ if } y_{k-1i} = 1 \]

This derivative can be computed using the sum rule, the product rule, and the derivative of a logistic regression equation. Using the sum rule, the first derivative can be written as:

\[ \frac{\partial (f(SES) + g(SES))}{\partial SES} = \frac{\partial f(SES)}{\partial SES} + \frac{\partial g(SES)}{\partial SES} \]

\[ \frac{\partial (f(SES) \times g(SES))}{\partial SES} = \frac{\partial f(SES)}{\partial SES} g(SES) + \frac{\partial g(SES)}{\partial SES} f(SES) \]

A special case occurs when a function of \( SES \) is multiplied by a constant \( c \) because the first derivative of a constant is zero:

\[ \frac{\partial (cf(SES))}{\partial SES} = \frac{\partial f(SES)}{\partial SES} c + \frac{\partial c}{\partial SES} f(SES) = \frac{\partial f(SES)}{\partial SES} c \]

\[ \text{Equation (1) is a logistic regression equation, which has a known first derivative (e.g. equation 3.14 Long, 1997):} \]
\[
\frac{\partial E(L)}{\partial \text{SES}} = \frac{\partial (1 - p_1)l_0}{\partial \text{SES}} + \frac{\partial p_1(1 - p_2)l_1}{\partial \text{SES}} + \frac{\partial p_1p_2(1 - p_3)l_2}{\partial \text{SES}} + \frac{\partial p_1p_2p_3l_3}{\partial \text{SES}}
\]

The notation was simplified by dropping the \(i\) subscript. Using the product rule, this can be rewritten as:

\[
\frac{\partial E(L)}{\partial \text{SES}} = l_0 \frac{\partial (1 - p_1)}{\partial \text{SES}} + \\
l_1 \left( \frac{\partial p_1}{\partial \text{SES}}(1 - p_2) + \frac{\partial (1 - p_2)}{\partial \text{SES}}p_1 \right) + \\
l_2 \left( \frac{\partial p_1}{\partial \text{SES}}p_2(1 - p_3) + \frac{\partial p_2}{\partial \text{SES}}p_1(1 - p_3) + \frac{\partial (1 - p_3)}{\partial \text{SES}}p_1p_2 \right) + \\
l_3 \left( \frac{\partial p_1}{\partial \text{SES}}p_2p_3 + \frac{\partial p_2}{\partial \text{SES}}p_1p_3 + \frac{\partial p_3}{\partial \text{SES}}p_1p_2 \right)
\]

All derivatives that are in the equation are derivatives of logistic regression equations. To facilitate the comparison with the previous equation, curly brackets are used to enclose the derivatives:

\[
\frac{\partial E(L)}{\partial \text{SES}} = l_0\{-p_1(1 - p_1)\lambda_1\} + \\
l_1 \{(p_1(1 - p_1)\lambda_1)(1 - p_2) + \{-p_2(1 - p_2)\lambda_2\}p_1\} + \\
l_2 \{(p_1(1 - p_1)\lambda_1)p_2(1 - p_3) + \{p_2(1 - p_2)\lambda_2\}p_1(1 - p_3) + \{-p_3(1 - p_3)\lambda_3\}p_1p_2\} + \\
l_3 \{(p_1(1 - p_1)\lambda_1)p_2p_3 + \{p_2(1 - p_2)\lambda_2\}p_1p_3 + \{p_3(1 - p_3)\lambda_3\}p_1p_2\}
\]

\[
\frac{\partial p_k}{\partial \text{SES}} = p_{k_i}(1 - p_{k_i})\lambda_k
\]

Together with the sum and the product rule this also implies that:

\[
\frac{\partial (1 - p_{k_i})}{\partial \text{SES}} = \frac{\partial 1}{\partial \text{SES}} + \frac{\partial - p_{k_i}}{\text{SES}} \quad \text{(sum rule)}
\]

\[
= \frac{\partial p_{k_i}}{\text{SES}} \quad \text{(product rule)}
\]

\[
= -p_{k_i}(1 - p_{k_i})\lambda_k
\]

24
The terms in this equation can be rearranged in such a way that all elements that have the same IEOpp ($\lambda$) in common are grouped together.

$$\frac{\partial E(L)}{\partial SES} = \{-p_1(1-p_1)l_0 + p_1(1-p_1)(1-p_2)l_1 + p_1(1-p_1)p_2(1-p_3)l_2 + p_1(1-p_1)p_2p_3l_3\} \lambda_1 +$$
$$\{-p_2(1-p_2)p_1l_1 + p_2(1-p_2)p_1(1-p_3)l_2 + p_2(1-p_2)p_1p_3l_3\} \lambda_2 +$$
$$\{-p_3(1-p_3)p_1p_2l_2 + p_3(1-p_3)p_1p_2l_3\} \lambda_3$$

Simplifying this equation will yield equation (3):

$$\frac{\partial E(L)}{\partial SES} =$$
$$\{1 \times p_1(1-p_1) \times [(1-p_2)l_1 + p_2(1-p_3)l_2 + p_2p_3l_3 - l_0] \} \lambda_1 +$$
$$\{p_1 \times p_2(1-p_2) \times [(1-p_3)l_2 + p_3l_3 - l_1] \} \lambda_2 +$$
$$\{p_1p_2 \times p_3(1-p_3) \times [(l_3 - l_2)] \} \lambda_3$$

B Scaling levels of education

To estimate the scale of education, this paper assumes that when estimating the effect of education on income, one does not need to a priori fix the scale of education; one can simply add education as a series of dummies. One way to interpret this model is that it simultaneously estimates the metric and the effect of education. This feature will be used to estimate a scale for the levels of education. Such a model is shown in equation (6), in which five diplomas are distinguished: $LO$, $LBO/MAVO$, $HAVO/VWO$, $MBO$, and $HBO/WO$.

$$\ln(inc) = \beta_0 + \beta_1 LO + \beta_2 LBO/MAVO + \beta_3 HAVO/VWO + \beta_4 MBO + \beta_5 HBO/WO + \cdots$$

(6)

In order for this to be identified, $\beta_1$ is constrained to be 0, in other words, the dummy for primary education is left out. A scale of educational levels will measure the relative distances
between diplomas. So, if the value of LO is fixed to 0 and that of HBO/WO to 1, then the
scaling will assign positions to all other diplomas relative to these two diplomas. These two
constraints will fix the origin at LO and the unit at the distance between LO and HBO/WO.
This new variable can be written like equation (7):

\[ ed = \alpha_1 LO + \alpha_2 LBO/MAVO + \alpha_3 HAVO/VWO + \alpha_4 MBO + \alpha_5 HBO/WO \] (7)

A person with only primary education gets \( \alpha_1 \), a person with LBO or MAVO gets \( \alpha_2 \), etc. In other words, the \( \alpha_\)s form the scale. If the effect of this scaled education is called \( \gamma_1 \), then the effect of education on income can be written like equation (8).

\[ \ln(inc) = \beta_0 + \gamma_1 ed + \cdots = \beta_0 + \gamma_1 (\alpha_1 LO + \alpha_2 LBO/MAVO + \alpha_3 HAVO/VWO + \alpha_4 MBO + \alpha_5 HBO/WO) \] (8)

All parameters in model (8) can be calculated from the parameters in model (6). The relationship between the parameters in the two models is given below:

\[ \gamma_1 = \beta_5 \]
\[ \alpha_1 = 0 \]
\[ \alpha_2 = \frac{\beta_2}{\beta_5} \]
\[ \alpha_3 = \frac{\beta_3}{\beta_5} \]
\[ \alpha_4 = \frac{\beta_4}{\beta_5} \]
\[ \alpha_5 = 1 \]

Model (8) is thus just a reparameterization of model (6). This relationship becomes more complicated when one adds interactions with education. One important interaction is the
interaction with time: over time, one might expect to need more education to achieve the same income. If one assumes a scaling of education that is constant over time, then this implies the constraint that the relative distances between diplomas remain the same. Such a model was estimated for men only using data from the ISMF that contain information about the income at the time of the interview. To allow for inflation and the change in currency (Dutch guilders to euros), these incomes were transformed to represent income in terms of euros from 2000. As is common with income data, the natural logarithm is taken allow for the fact that income tends to be right skewed. Income is explained using the following variables:

- education (measured in a scale that is to be estimated)
- age and age squared (in tens of years and zero when someone is 40),
- year (in tens of years and zero in 1958, the earliest survey) added as three splines with knots in 1975 and 1990,
- the father’s occupational status (in ISEI points, re-scaled to range between 0 and 1),
- interaction of the father’s occupational status with year splines, and
- interaction of education with year splines.

The changing effect of education over time is shown in the $\gamma$ panel in table 4. It shows that in 1958, someone with an HBO/WO education earned about 47% more than someone with primary education. This remained constant between 1958 and 1975, decreased by about 17 percentage points per 10 years between 1975 and 1990, and increased by about 19 percentage points per 10 years between 1990 and 2005. However, the scaling is constrained to remain constant (the $\alpha$ panel in table 4). This constraint was tested and rejected, but this is not surprising given the large sample size (31,253 respondents). However, the difference in BIC scores is 26.09 points smaller for the constrained model, indicating ‘very strong’ (Raftery, 1995) or ‘decisive’ (Jeffreys, 1961) evidence in favour of the model with constant scaling.

---

7The surveys used to estimate the scaling of education are: net58, net67t, net70, net71, net71c, net74p, net76j, net77, net77e, net79p, net81e, net82c, net82u, net85o, net86e, net86i, net87l, net87j, net87s, net88o, net90, net90o, net91j, net92f, net92o, net92t, net94e, net94h, net94o, net95s, net96, net96c, net96o, net96y, net98, net98e, net98o, net99, net99i, next0s, next02e, next03, next04i. Codes refer to (Ganzeboom, 2005).
of education over time. The overall conclusion is thus that the relative distances between
diplomas remained unchanged between 1958 and 2005.

[Table 4 about here.]

An alternative method of scaling education would be to look at the official number of years
needed to obtain a diploma. The two metrics are compared in figure 11, by transforming both
to the same metric: the mean level of education for the cohort which is 12 in 1970 is fixed
to 0 and the standard deviation for that same cohort is fixed to 1. The two main differences
are that 1) MBO and HAVO/VWO have changed place in such a way that in the estimated
scale, MBO is less valuable then HAVO/VWO; and 2) the distance between HBO/WO and
the rest is larger in the estimated scale. These two scales measure different things, namely:
education measured in the number of years officially needed to obtain the diploma represents
the investment, and the estimated scale measures a resource — an enhanced ability to earn
income.

[Figure 11 about here.]

References


York: Wiley.

vanaf de Middeleeuwen tot aan de huidige tijd.* Assen: Van Gorcum.


van Essen, M. 1990. *Opvoeden met een dubbel doel: twee eeuwen meisjesonderwijs in Nederland*. Amsterdam: SUA.
Figure 1: Hypothetical educational system

\[
\begin{align*}
 l_0 &= 0 \\
 l_1 &= 6 \\
 l_2 &= 12 \\
 l_3 &= 16
\end{align*}
\]
Figure 2: The Dutch education system

LO (primary) → HAVO (senior general secondary) → VWO (pre-university) → WO (university)

LO (primary) → MAVO (junior general secondary) → MBO (senior secondary vocational)

LO (primary) → LBO (junior vocational) → HAVO (senior general secondary) → HBO (higher professional) → WO (university)
Figure 3: Simplified model of the Dutch education system

- LO
  - $p_1$: continue
  - $1 - p_1$: exit

- $1 - p_2$: LBO/MAVO
  - $1 - p_3$: exit
  - $p_3$: MBO
    - $l_3 = -0.17$

- $p_2$: HAVO/VWO
  - $1 - p_4$: exit
  - $p_4$: HBO/WO
    - $l_5 = 1.35$

- $1 - p_4$: exit

- $l_1 = -2.10$
- $l_2 = -0.71$
- $l_3 = -0.17$
- $l_4 = 0.10$
Figure 4: Distribution of highest achieved level of education for men and women over cohorts
Figure 5: Expected level of education according to the sequential response model
Figure 6: IEOut according to the sequential response model.
Figure 7: Decomposition of IEOut into IEOpps and weights

The diagram illustrates the decomposition of IEOut into IEOpps and weights over different years (1900 to 1975). It shows log odds ratio and weight distributions for various educational levels (HAVO/VWO, HBO/WO, LBO/MAVO, MBO) and a category labeled 'LO v more'. The bars represent the weight distribution for each year, with different educational levels indicated by different colors and patterns.
Figure 8: Decomposition of IEOut into IEOpps and weights
Figure 9: Decomposition of weights into ‘at risk’, ‘variance’, and ‘gain’
Figure 10: Decomposition of weights into ‘at risk’, ‘variance’, and ‘gain’
Figure 11: Estimated scale of education versus a scale based on years of education

standardized levels of education
mean = 0, sd = 1 for the cohort born in 1940
Table 1: Levels of education in the Netherlands (Boekholt and de Booy, 1987; UNESCO, 1997)

<table>
<thead>
<tr>
<th>Dutch name</th>
<th>English name</th>
<th>years(^\d)</th>
<th>ISCED</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>Primary</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>LBO</td>
<td>Junior vocational</td>
<td>10</td>
<td>2C</td>
</tr>
<tr>
<td>MAVO</td>
<td>Junior general secondary</td>
<td>9 / 10</td>
<td>2B(^\d)</td>
</tr>
<tr>
<td>MBO</td>
<td>Senior secondary vocational</td>
<td>14</td>
<td>3C</td>
</tr>
<tr>
<td>HAVO</td>
<td>Senior general secondary</td>
<td>11</td>
<td>3B(^\d)</td>
</tr>
<tr>
<td>VWO</td>
<td>Pre-university</td>
<td>12</td>
<td>3A</td>
</tr>
<tr>
<td>HBO</td>
<td>Higher professional</td>
<td>15</td>
<td>5B</td>
</tr>
<tr>
<td>WO</td>
<td>University</td>
<td>16</td>
<td>5A</td>
</tr>
</tbody>
</table>

\(^\d\) Years refer to the situation after 1968
\(^\d\) These levels were originally intended to be terminal levels of education for most students (so 2C or 3C) but evolved into levels that primarily grant access to subsequent levels of education.
<table>
<thead>
<tr>
<th></th>
<th>LO vs more</th>
<th>LBO/MAVO vs HAVO/VWO</th>
<th>LBO/MAVO vs MBO</th>
<th>HAVO/VWO vs HBO/WO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Father’s status</td>
<td>0.973</td>
<td>0.595</td>
<td>0.223</td>
<td>0.320</td>
</tr>
<tr>
<td></td>
<td>(15.87)</td>
<td>(12.16)</td>
<td>(2.37)</td>
<td>(4.35)</td>
</tr>
<tr>
<td>Father’s status X cohort</td>
<td>-0.074</td>
<td>0.006</td>
<td>0.011</td>
<td>-0.016</td>
</tr>
<tr>
<td></td>
<td>(-5.17)</td>
<td>(0.59)</td>
<td>(0.61)</td>
<td>(-1.13)</td>
</tr>
<tr>
<td>Cohort</td>
<td>0.557</td>
<td>0.244</td>
<td>0.563</td>
<td>0.357</td>
</tr>
<tr>
<td></td>
<td>(23.36)</td>
<td>(11.20)</td>
<td>(13.59)</td>
<td>(9.89)</td>
</tr>
<tr>
<td>Cohort1</td>
<td>0.001</td>
<td>0.020</td>
<td>-0.001</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(8.84)</td>
<td>(-0.32)</td>
<td>(4.90)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.208</td>
<td>-0.968</td>
<td>-3.750</td>
<td>-0.357</td>
</tr>
<tr>
<td></td>
<td>(-2.68)</td>
<td>(-12.32)</td>
<td>(-23.71)</td>
<td>(-2.75)</td>
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</tbody>
</table>

| N                           | 43539      |
| Log likelihood              | -48889.247|

* Z statistics in parentheses
<table>
<thead>
<tr>
<th></th>
<th>LO v more</th>
<th>LBO/MAVO v HAVO/VWO</th>
<th>LBO/MAVO v MBO</th>
<th>HAVO/VWO v HBO/WO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Father’s status</td>
<td>0.971</td>
<td>0.947</td>
<td>0.317</td>
<td>-0.114</td>
</tr>
<tr>
<td></td>
<td>(16.91)</td>
<td>(16.14)</td>
<td>(3.32)</td>
<td>(-1.27)</td>
</tr>
<tr>
<td>Father’s status X cohort</td>
<td>-0.083</td>
<td>-0.051</td>
<td>-0.003</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td>(-6.14)</td>
<td>(-4.62)</td>
<td>(-0.16)</td>
<td>(3.34)</td>
</tr>
<tr>
<td>Cohort</td>
<td>0.729</td>
<td>0.215</td>
<td>0.367</td>
<td>0.288</td>
</tr>
<tr>
<td></td>
<td>(30.59)</td>
<td>(7.43)</td>
<td>(8.24)</td>
<td>(6.14)</td>
</tr>
<tr>
<td>Cohort$_1$</td>
<td>0.001</td>
<td>-0.004</td>
<td>-0.033</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>(0.27)</td>
<td>(-1.60)</td>
<td>(-8.31)</td>
<td>(3.09)</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.283</td>
<td>-1.708</td>
<td>-3.482</td>
<td>-0.297</td>
</tr>
<tr>
<td></td>
<td>(-16.53)</td>
<td>(-15.58)</td>
<td>(-20.17)</td>
<td>(-1.66)</td>
</tr>
<tr>
<td>N</td>
<td>43139</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Log likelihood</td>
<td>-44457.068</td>
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z statistics in parentheses
### Table 4: Scaling of education

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<tr>
<th>α</th>
<th>b</th>
<th>z</th>
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<tbody>
<tr>
<td>LO</td>
<td>0</td>
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</tr>
<tr>
<td>LBO/MAVO</td>
<td>0.395</td>
<td>(21.91)</td>
</tr>
<tr>
<td>MBO</td>
<td>0.549</td>
<td>(19.21)</td>
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<tr>
<td>HAVO/VWO</td>
<td>0.667</td>
<td>(24.65)</td>
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<tr>
<td>HBO/WO</td>
<td>1</td>
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<table>
<thead>
<tr>
<th>γ</th>
<th>b</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>-0.0868</td>
<td>(-2.41)</td>
</tr>
<tr>
<td>Year₁</td>
<td>0.0707</td>
<td>(1.67)</td>
</tr>
<tr>
<td>Year₂</td>
<td>-0.115</td>
<td>(-2.53)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.643</td>
<td>(12.09)</td>
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</table>

<table>
<thead>
<tr>
<th>β</th>
<th>b</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>0.115</td>
<td>(25.28)</td>
</tr>
<tr>
<td>Age²</td>
<td>-0.0715</td>
<td>(-20.19)</td>
</tr>
<tr>
<td>Fisei</td>
<td>0.476</td>
<td>(5.47)</td>
</tr>
<tr>
<td>Fiseixyear</td>
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<td>(-1.36)</td>
</tr>
<tr>
<td>Fiseixyear₁</td>
<td>0.0560</td>
<td>(0.78)</td>
</tr>
<tr>
<td>Fiseixyear₂</td>
<td>-0.0812</td>
<td>(-1.08)</td>
</tr>
<tr>
<td>Year</td>
<td>0.833</td>
<td>(34.88)</td>
</tr>
<tr>
<td>Year₁</td>
<td>0.287</td>
<td>(9.07)</td>
</tr>
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<td>Year₂</td>
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<td>(-5.53)</td>
</tr>
<tr>
<td>Constant</td>
<td>5.058</td>
<td>(153.45)</td>
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</table>