Direct and indirect effects in a `logit` model

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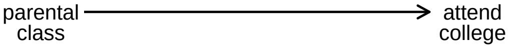
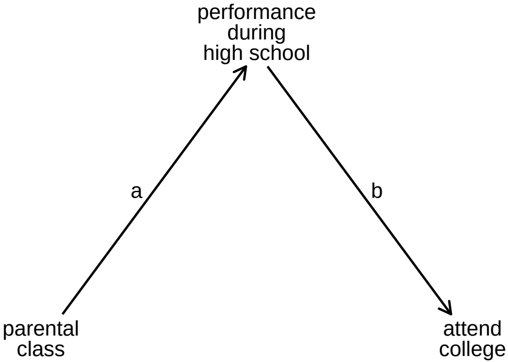
March 27, 2008

Outline

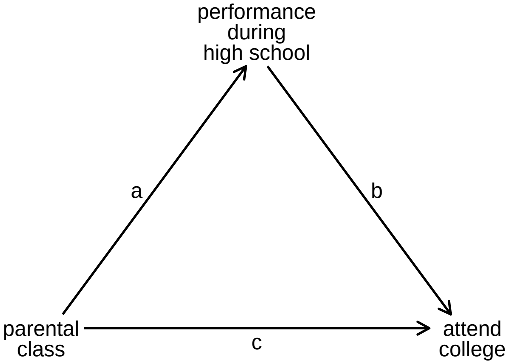
The Total Effect



The Indirect Effect

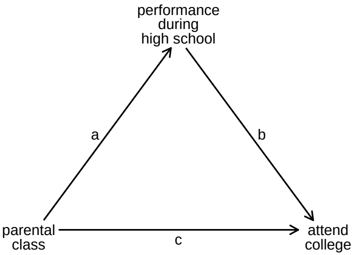


The Direct Effect



The aim

The aim is to find the size of the indirect effect relative to the direct effect.



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Estimation

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  1. college = class + performance
  2. college = class

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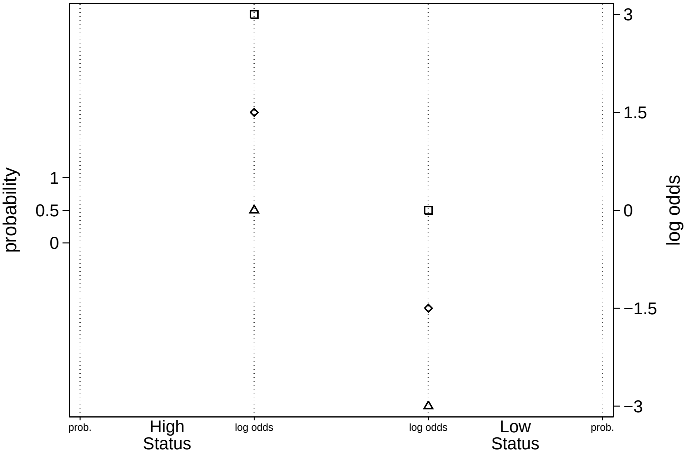
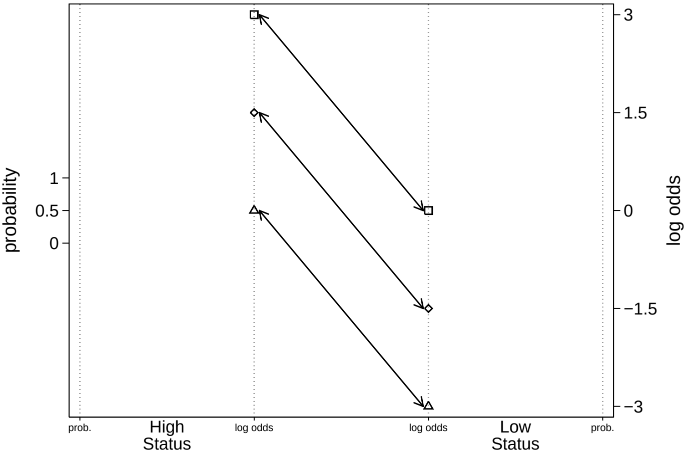
- The direct effect is the effect of class in model 1.
- The total effect is the effect of class in model 2.
- The indirect effect is the total effect - direct effect.
- This won't work when using `logit`

Log odds


probability
1
0.5
0
log odds
3
1.5
0
−1.5
−3
prob.
High
Status
log odds
log odds
Low
Status
prob.

Effect


probability
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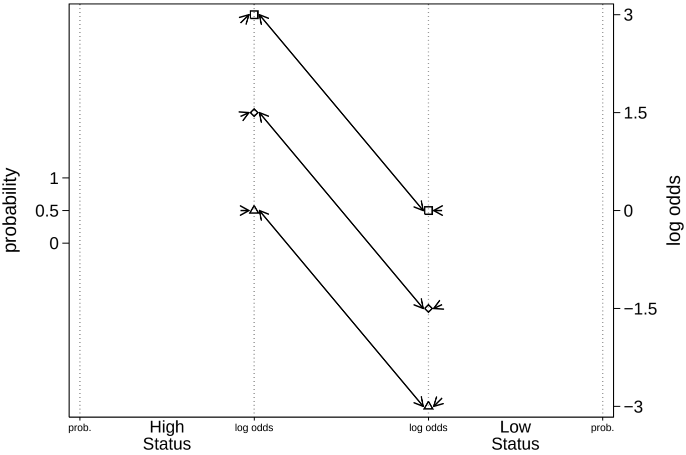
log odds
prob.
High Status
log odds
log odds
Low Status
prob.

Transform log odds to probability



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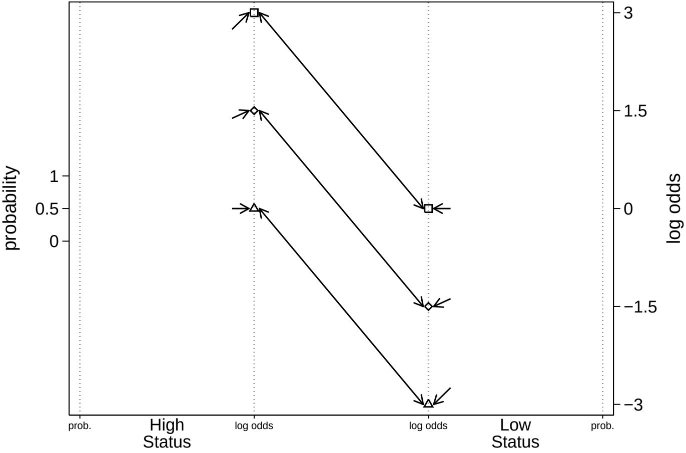

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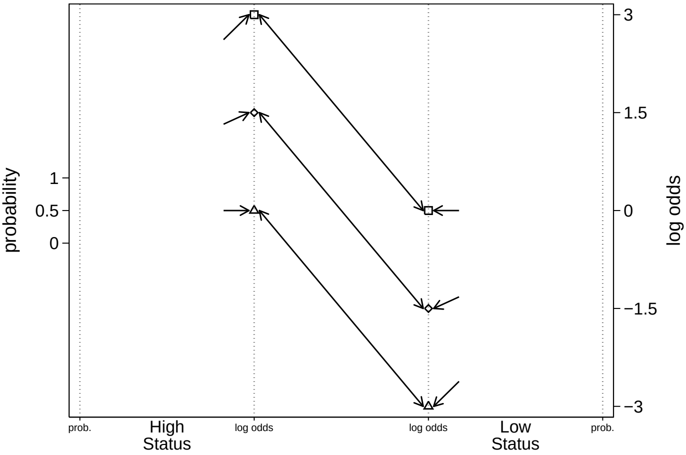

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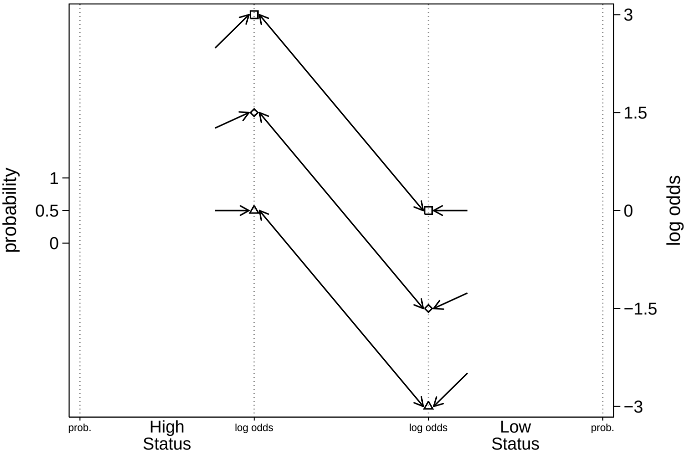

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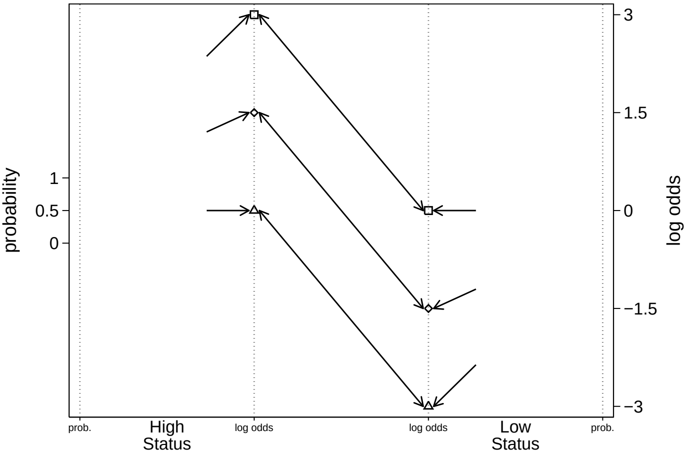

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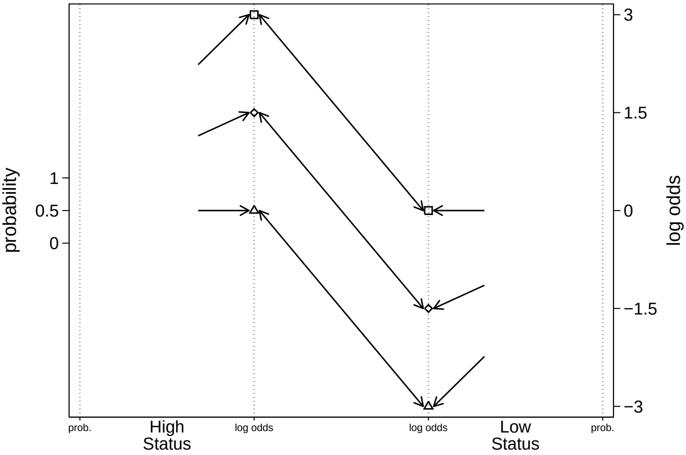

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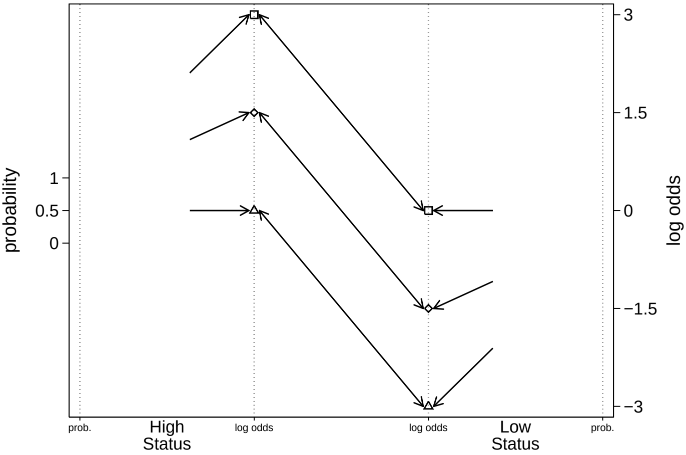

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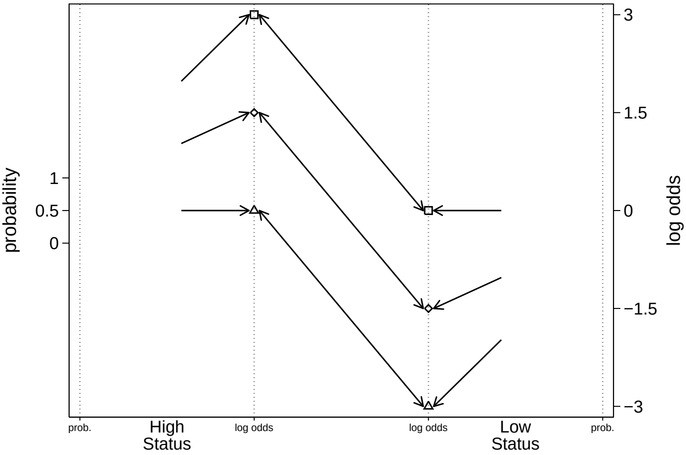

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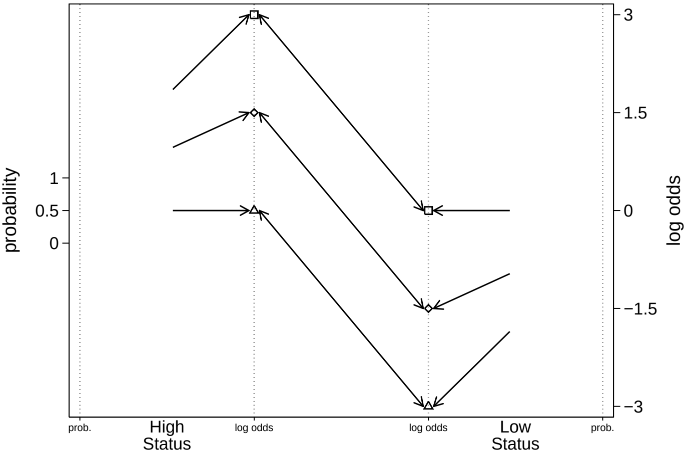

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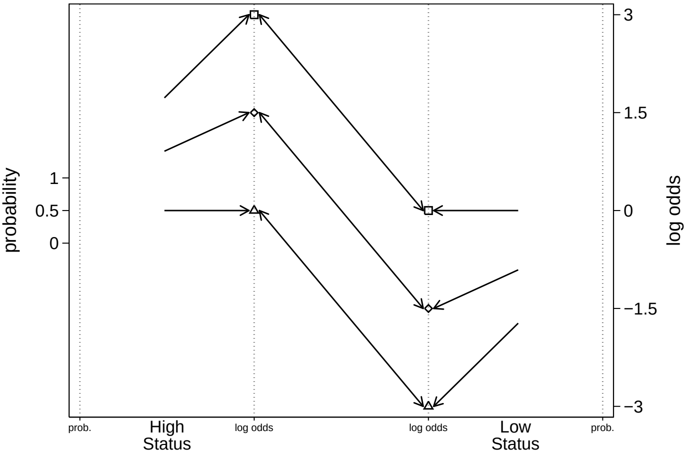

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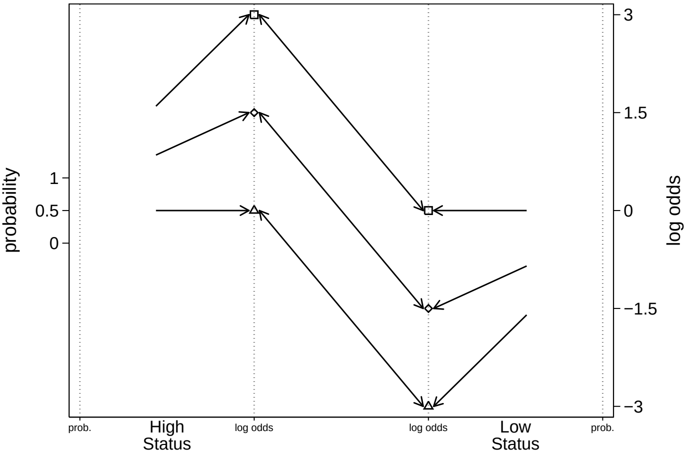

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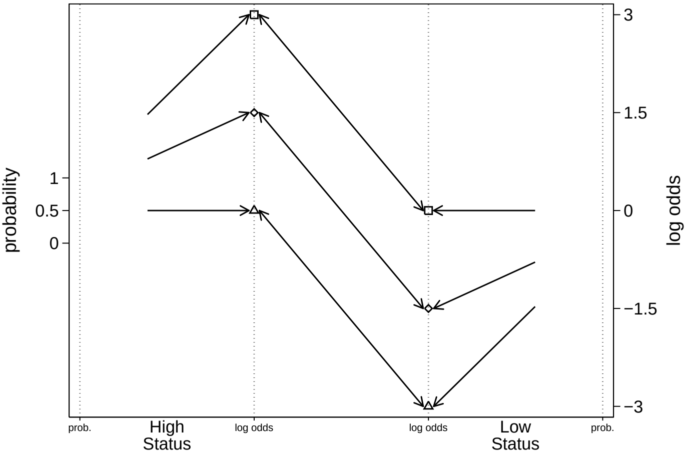

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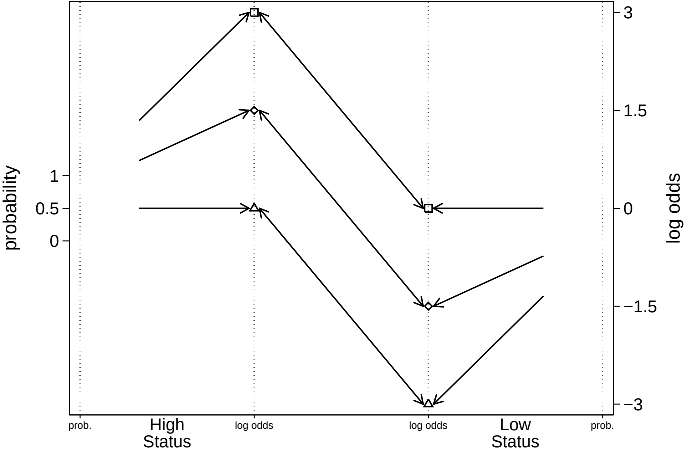

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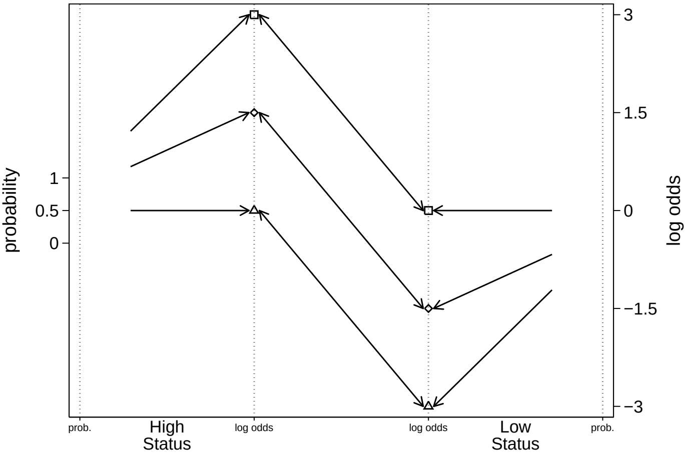

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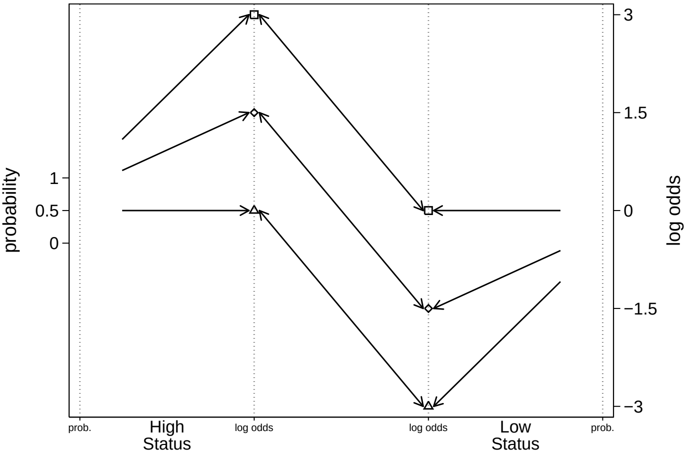

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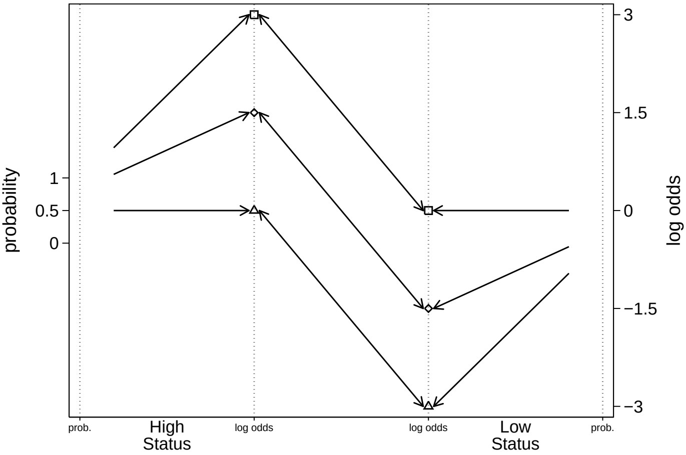

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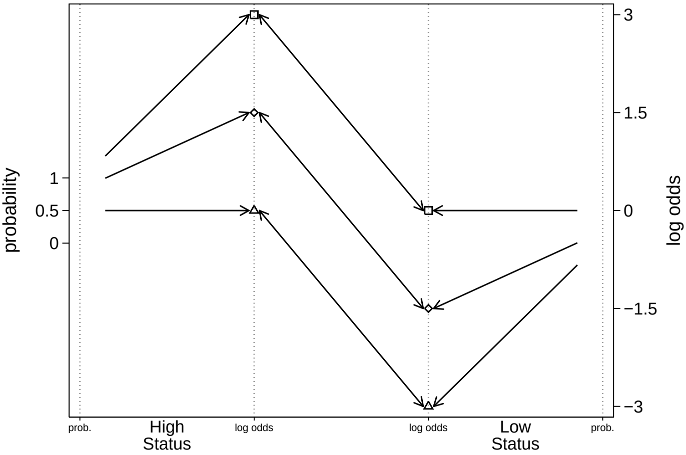

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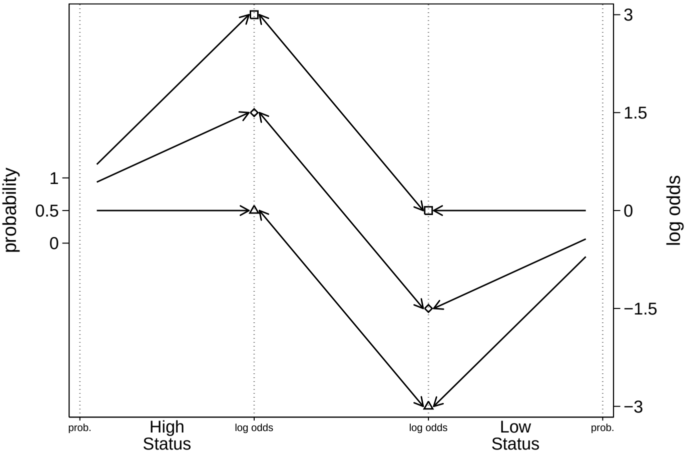

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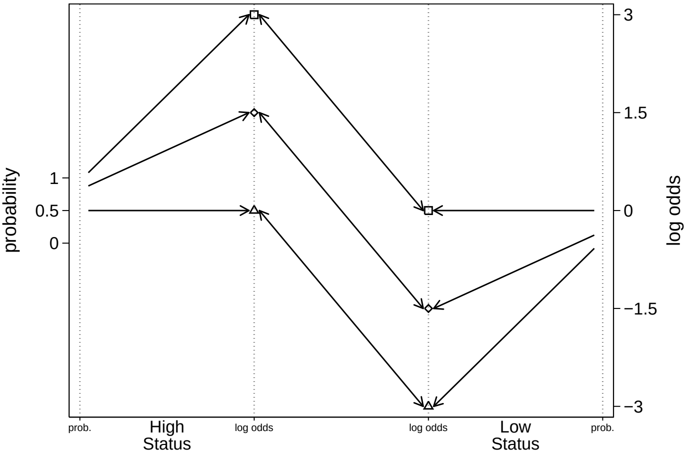

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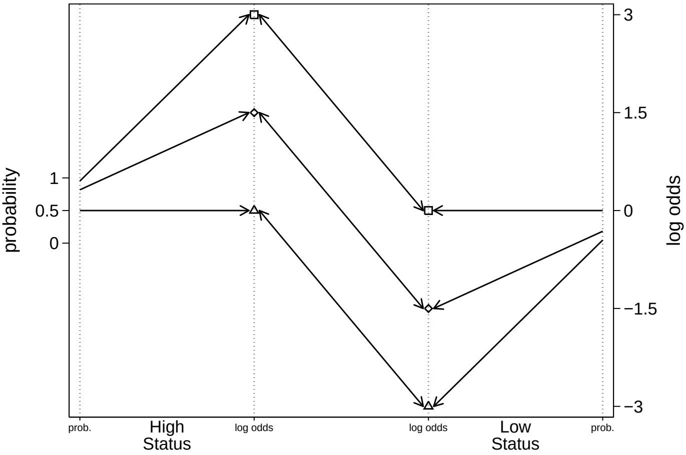

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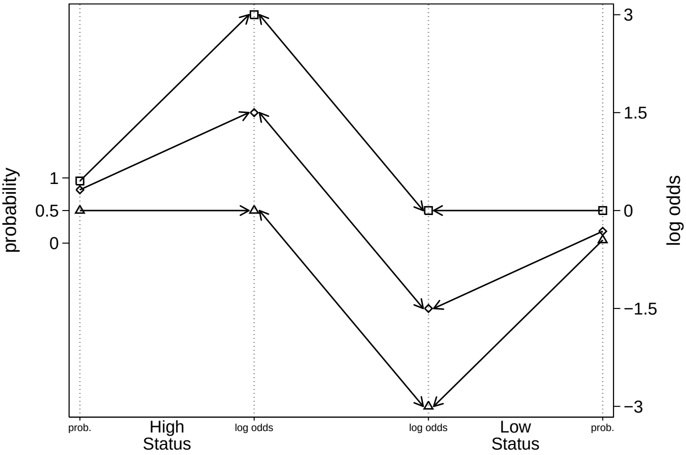
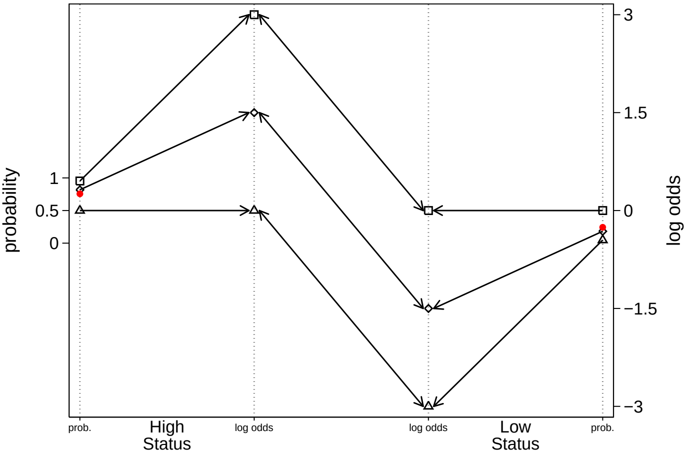
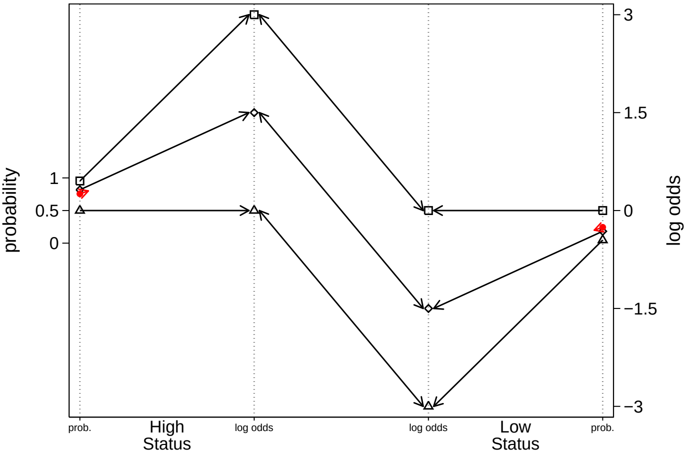


Average probability



transforming the average probability



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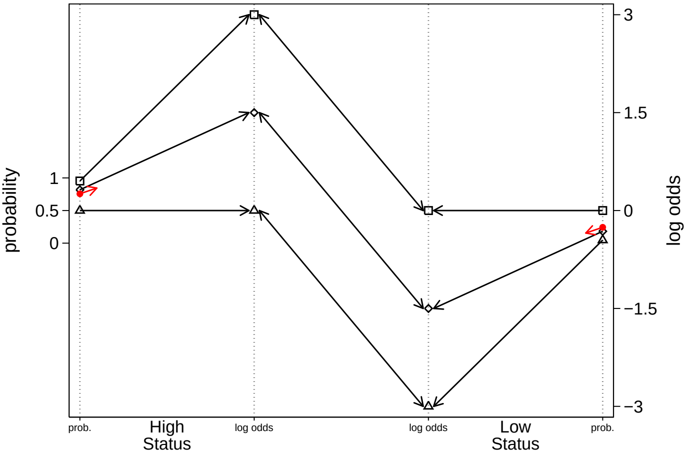

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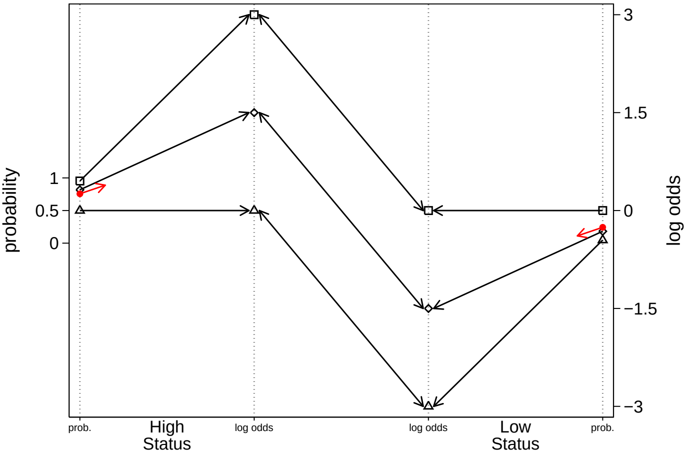

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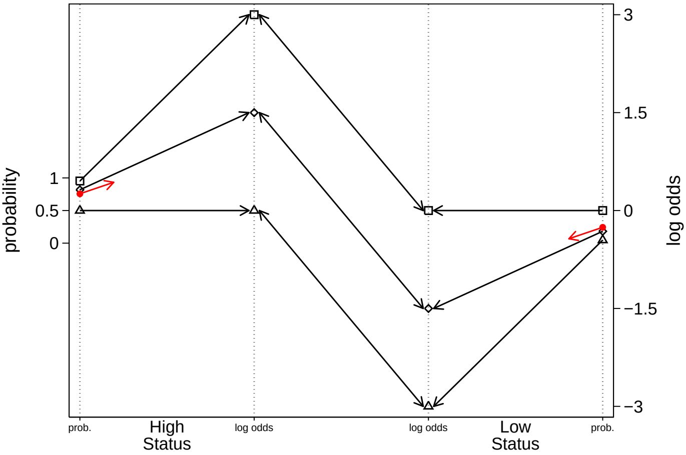

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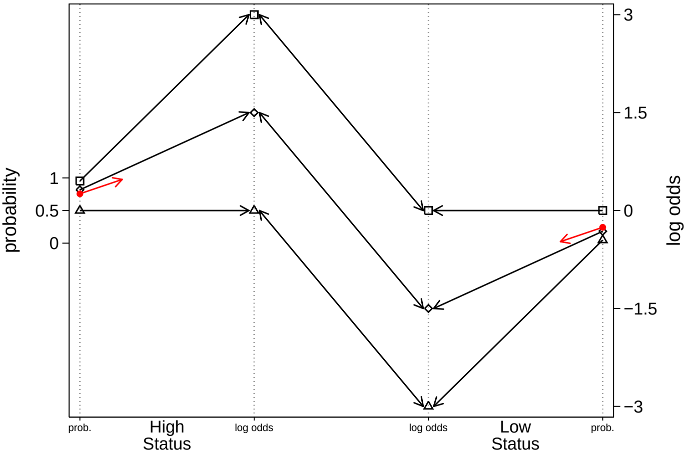

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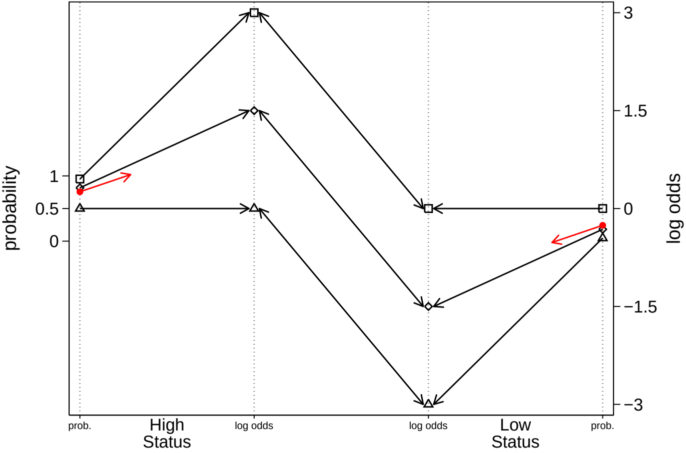

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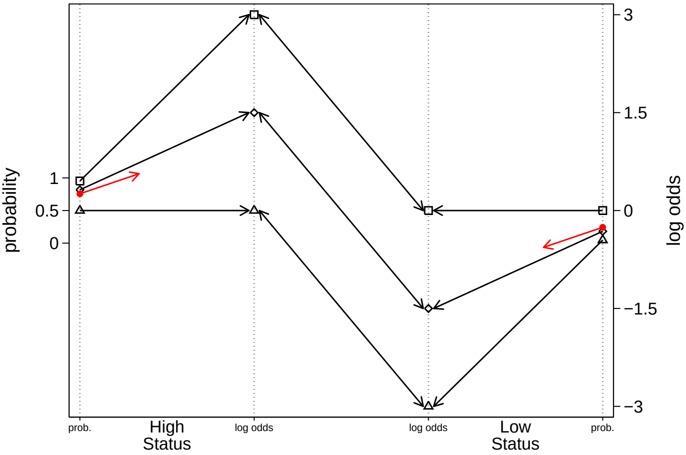

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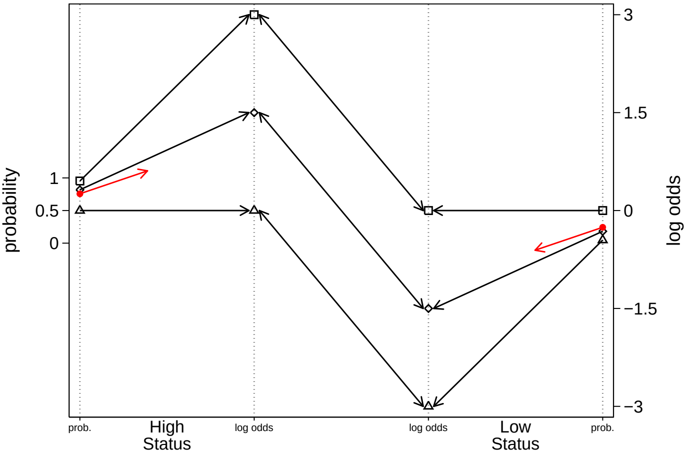

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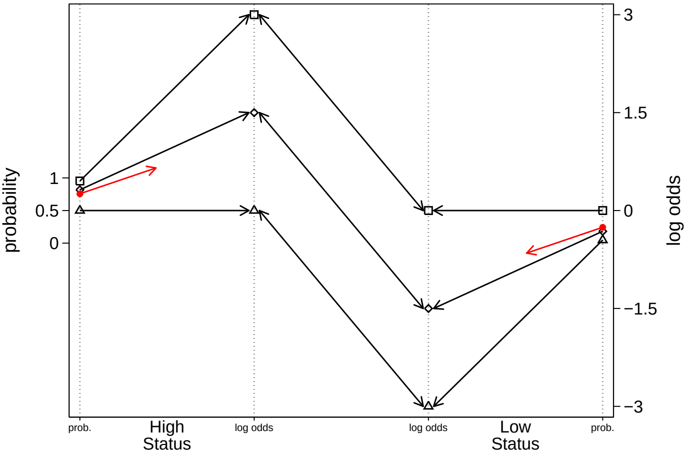

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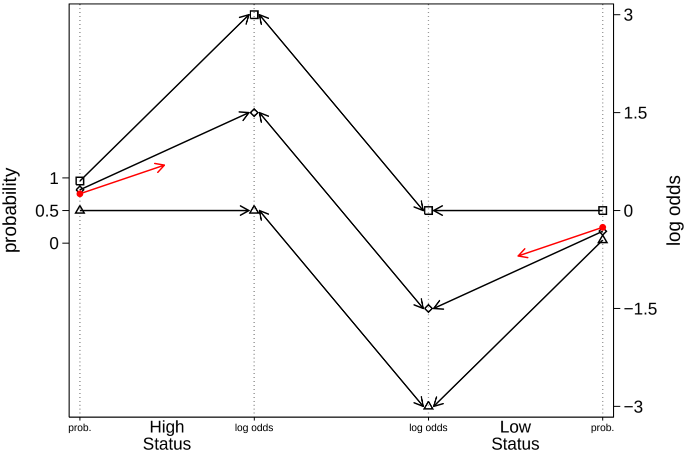

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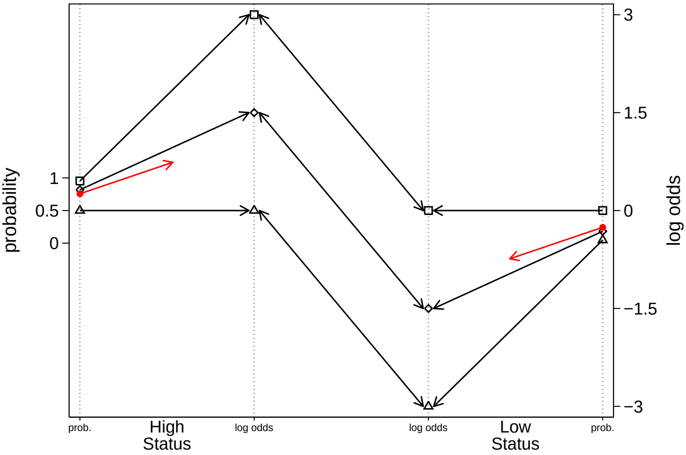

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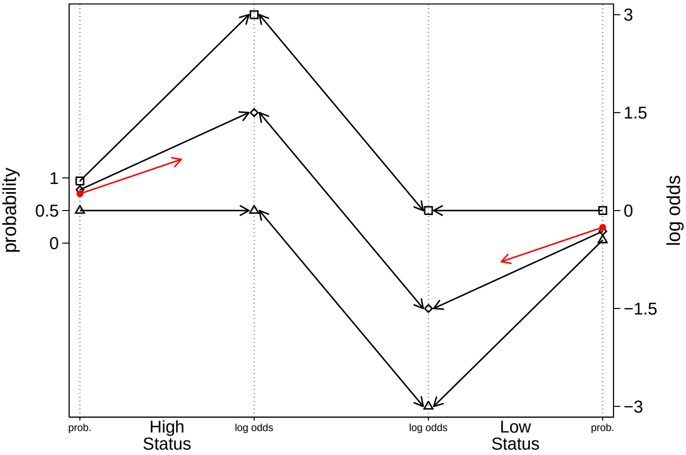

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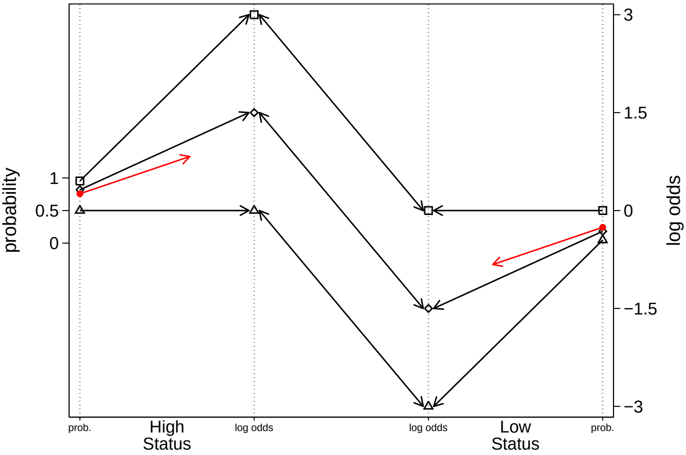

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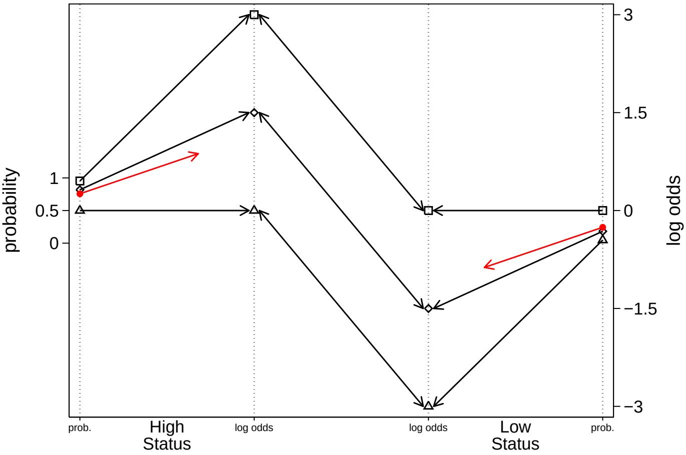

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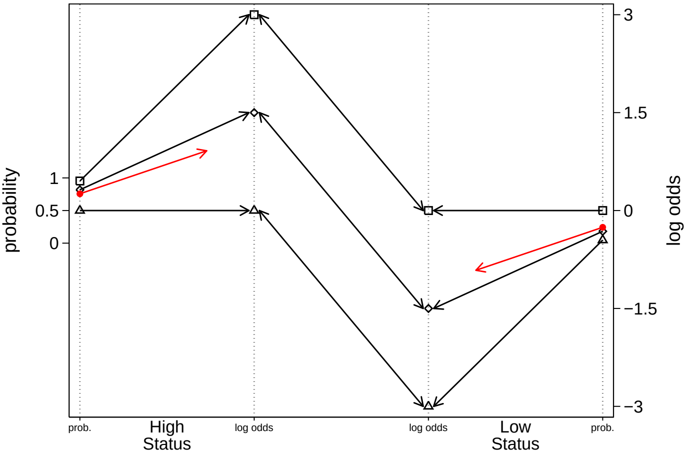

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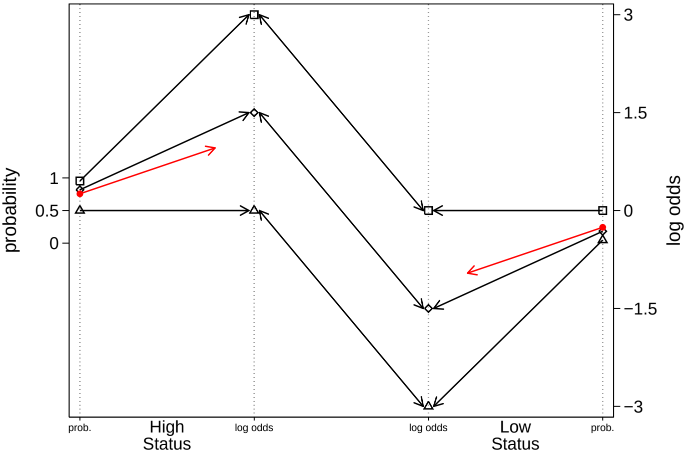

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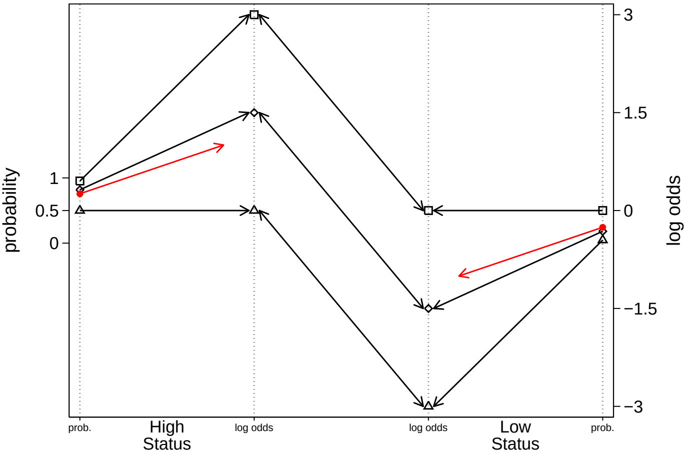

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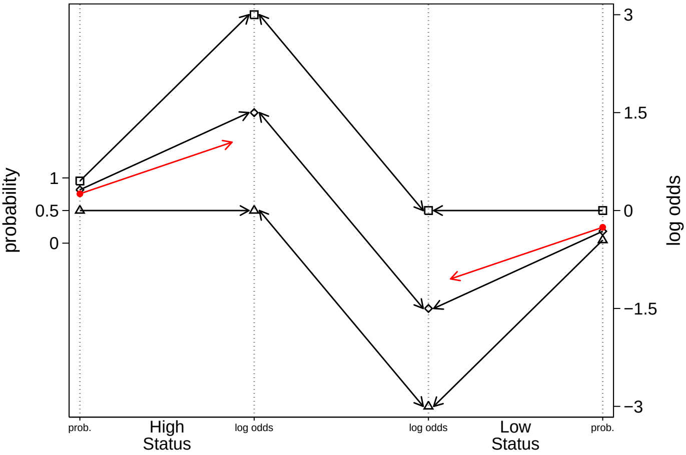

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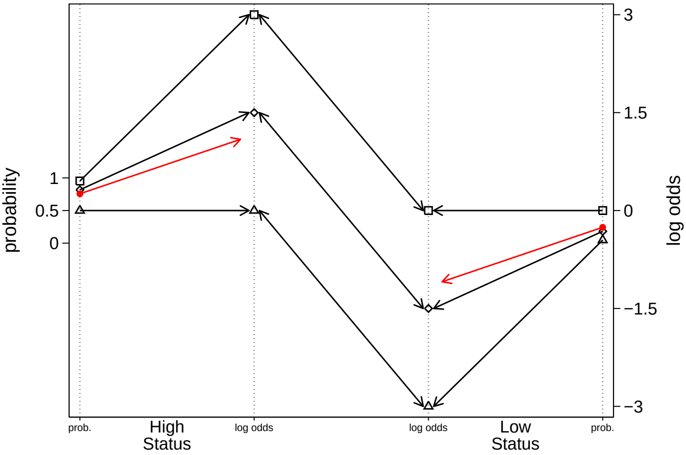

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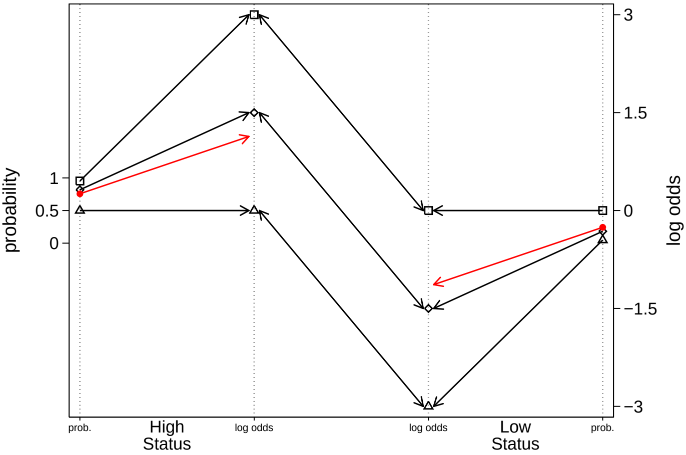

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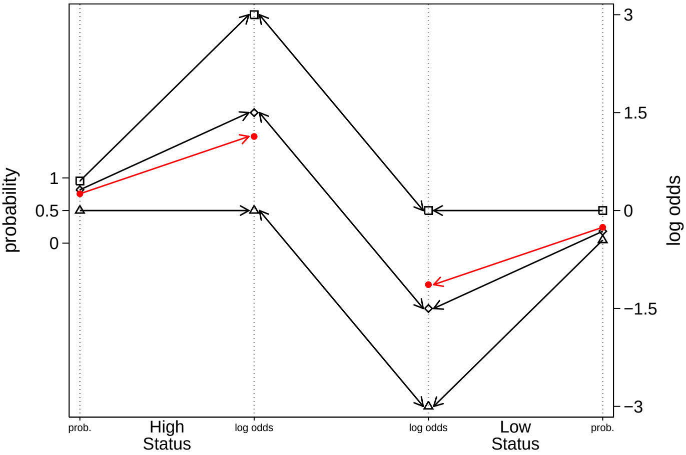
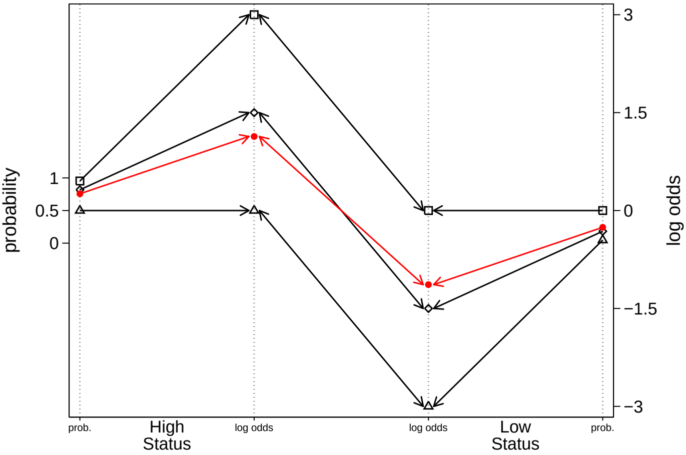


Effect of class without controlling for performance



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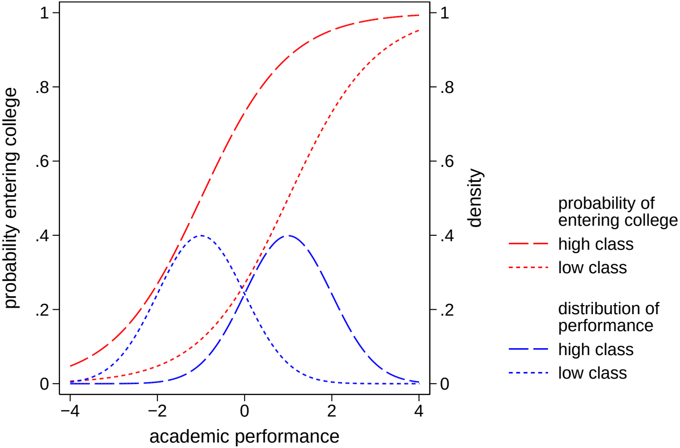

Counterfactual odds

		logit curve	
		low class	high class
distribution of	low class	O_{ll}	O_{lh}
performance	high class	O_{hl}	O_{hh}

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$$
\begin{aligned}
\ln\left(\frac{O_{hl}}{O_{ll}}\right) + \ln\left(\frac{O_{hh}}{O_{hl}}\right) &= \ln\left(\frac{O_{hh}}{O_{ll}}\right) \\
\underbrace{\ln\left(\frac{O_{hh}}{O_{lh}}\right)}_{\textit{indirect}} + \underbrace{\ln\left(\frac{O_{lh}}{O_{ll}}\right)}_{\textit{direct}} &= \underbrace{\ln\left(\frac{O_{hh}}{O_{ll}}\right)}_{\textit{total}}
\end{aligned}
$$

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As proposed by Buis (2008):

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As proposed by Erikson and colleagues (2005):

- $\int_{-\infty}^{\infty} \phi(\mu_{c_1}, \sigma_{c_1}) \Lambda(\alpha_{c_2} + \beta_{c_2} x) dx$

Outline

The aim

The problem

The solution

example

Descriptives

```
. table ocf57 if !missing(hsrankq, college) , ///
> contents(mean college mean hsrankq freq) format(%9.3g)
```

occupation of r father in 1957	mean(college)	mean(hsrankq)	Freq.
Unskilled	.287	46.4	3,528
Farming	.277	51.9	1,690
Skilled	.38	50.6	868
White collar	.54	54	1,868
Professional or executive	.771	60.5	969

The `ldecomp` package

`ldecomp` *depvar* [*weight*] [*if*] [*in*] , `direct(`*varname*`)` `indirect(`*varlist*`)` [`obspr predpr predodds rindirect rdirect lor noor normal range(`*##*`) nip(`*#*`) interactions`]

Counterfactual proportions

```
. ldecomp college , direct(ocf57) indirect(hsrankq) predpr predodds lor rind
```

predicted and counterfactual proportions

	association				
distribution	Unskilled	Farming	Skilled	White col~r	Professio~e
Unskilled	.287	.251	.356	.492	.702
Farming	.316	.277	.388	.527	.731
Skilled	.309	.271	.38	.518	.723
White collar	.329	.289	.402	.54	.74
Profession~e	.365	.323	.441	.581	.771

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distribution	Unskilled	Farming	association Skilled	White col~r	Professio~e
Unskilled	.287	.251	.356	.492	.702
Farming	.316	.277	.388	.527	.731
Skilled	.309	.271	.38	.518	.723
White collar	.329	.289	.402	.54	.74
Profession~e	.365	.323	.441	.581	.771

- The odds is $\frac{\textit{probability}}{1-\textit{probability}}$
- The odds that a child from a unskilled worker enters college is $\frac{.287}{1-.287} = .403$

Counterfactual odds

predicted and counterfactual odds

	association				
distribution	Unskilled	Farming	Skilled	White col~r	Professio~e
Unskilled	.403	.335	.553	.969	2.35
Farming	.462	.383	.635	1.12	2.72
Skilled	.447	.371	.613	1.07	2.61
White collar	.49	.407	.672	1.17	2.84
Profession~e	.574	.476	.789	1.38	3.36

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$$\underbrace{\frac{.574}{.403}}_{indirect} \times \underbrace{\frac{3.36}{.574}}_{direct} = \underbrace{\frac{3.36}{.403}}_{total}$$

Counterfactual odds

predicted and counterfactual odds

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$$\underbrace{1.43}_{\textit{indirect}} \times \underbrace{5.86}_{\textit{direct}} = \underbrace{8.35}_{\textit{total}}$$

Decomposition of odds ratios

decomposition of odds ratios

(method 1)

	indirect [i,j]/[j,j]	direct [i,i]/[i,j]	total [i,i]/[j,j]
2/1	1.15	.83	.951
3/1	1.11	1.37	1.52
4/1	1.22	2.4	2.92
5/1	1.43	5.86	8.35
3/2	.97	1.65	1.6
4/2	1.06	2.89	3.07
5/2	1.24	7.06	8.79
4/3	1.1	1.75	1.91
5/3	1.29	4.27	5.49
5/4	1.18	2.43	2.86

(method 2)

	indirect [i,i]/[j,i]	direct [j,i]/[j,j]	total [i,i]/[j,j]
2/1	1.14	.831	.951
3/1	1.11	1.37	1.52
4/1	1.21	2.4	2.92
5/1	1.43	5.85	8.35
3/2	.966	1.66	1.6
4/2	1.05	2.91	3.07
5/2	1.24	7.1	8.79
4/3	1.09	1.75	1.91
5/3	1.29	4.26	5.49
5/4	1.18	2.42	2.86

Column names:
i refers to the first category in the row name
j refers to the second category in the row name
first number in pair refers to the distribution
second number in pair refers to the association

value labels
1 Unskilled
2 Farming
3 Skilled
4 White collar
5 Professional or executive

Decomposition of log odds ratios

decomposition of log odds ratios

(method 1)

	indirect [i,j]/[j,j]	direct [i,i]/[i,j]	total [i,i]/[j,j]
2/1	.136	-.187	-.0504
3/1	.104	.316	.421
4/1	.196	.874	1.07
5/1	.354	1.77	2.12
3/2	-.0308	.502	.471
4/2	.0611	1.06	1.12
5/2	.218	1.95	2.17
4/3	.0911	.559	.65
5/3	.251	1.45	1.7
5/4	.164	.888	1.05

(method 2)

	indirect [i,i]/[j,i]	direct [j,i]/[j,j]	total [i,i]/[j,j]
2/1	.135	-.186	-.0504
3/1	.104	.317	.421
4/1	.193	.877	1.07
5/1	.357	1.77	2.12
3/2	-.0342	.505	.471
4/2	.0516	1.07	1.12
5/2	.213	1.96	2.17
4/3	.089	.561	.65
5/3	.253	1.45	1.7
5/4	.169	.884	1.05

Column names:
i refers to the first category in the row name
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value labels
1 Unskilled
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Relative importance of indirect effect

relative importance of indirect effect

	method 1	method 2	average
2/1	-2.7	-2.68	-2.69
3/1	.248	.247	.248
4/1	.183	.18	.182
5/1	.167	.168	.168
3/2	-.0654	-.0725	-.069
4/2	.0545	.0461	.0503
5/2	.101	.098	.0993
4/3	.14	.137	.139
5/3	.148	.149	.148
5/4	.156	.16	.158

value labels
1 Unskilled
2 Farming
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Discussion

This is “an area of active research”

Discussion

There are unanswered questions:

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- The need to take the average indirect effect is less than elegant.
- This procedure only provides a point estimate, and no tests or confidence intervals.
- How does it relate to the alternative method proposed by Fairlie (2005) and implemented by Ben Jann as the `fairlie` package?

References

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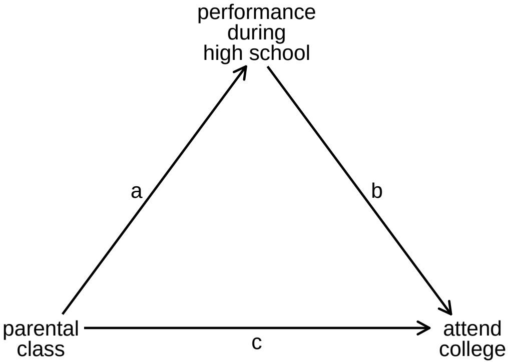
Intervening variable



Confounding variable